

Teaching for Understanding

by **Eric Milou, Ed.D.**

Building Understanding

It is very clear that effective mathematics instruction begins with effective teaching. But over the course of history, effective mathematics teaching has been defined in many ways. In the early half of the 20th century, proficiency was defined by facility with computation, while in the later half of the century, the standards-based movement emphasized problem solving and reasoning. Such debate has often been acrimonious and has led to many false beliefs about successful mathematics teaching. At the turn of the 21st century, however, the National Research Council published *Adding It Up: Helping Children Learn Mathematics* (NAP, 2001) in which it defined mathematical proficiency as having five interwoven components.

Conceptual Understanding

Conceptual understanding “reflects a student’s ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.”¹ With conceptual understanding, students are able to transfer their knowledge to new situations and contexts in order to solve the problem presented. It is this transfer of knowledge that is so vital for success not only in mathematics but in all disciplines and in the workplace. The authors of *Principles and Standards for School Mathematics* (NCTM, 2000) summarize it best: “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.”²

Procedural Fluency

In a position page on procedural fluency, the National Council of Teachers of Mathematics (NCTM) defines procedural fluency as “the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.”³

It should be noted that procedural fluency is more than memorizing procedures and facts. Procedural fluency builds on the foundation of conceptual understanding, so knowledge of procedures is no guarantee of conceptual understanding. For example, many secondary students learn to use the “FOIL” routine for the multiplication of binomials, without realizing that multiplying two binomials is a function of the distributive property.



Eric Milou

Eric Milou is an expert on using technology to increase student engagement and mathematical understanding. As an enVision Mathematics Grades 6-8 author, Dr. Milou contributes to ensuring vertical alignment to the enVision A|G|A program for Algebra 1, Geometry, and Algebra 2.

Acknowledgments

¹ NAEP (2003). What Does the NAEP Mathematics Assessment Measure? Online at nces.ed.gov/nationsreportcard/mathematics/abilities.asp.

² <http://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Principles,-Standards,-and-Expectations/>

³ <http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/>

Strategic Competence

Strategic competence is the ability to “formulate mathematical problems, represent them, and solve them.”⁴ While some may see this strand as similar to what has been called problem solving and problem formulation in mathematics education, it is important to point out that strategic competence involves authentic problem solving—problems for which students must formulate a mathematical model to represent the problem context and then determine the operations necessary to come up with a viable solution.

Learning to solve these authentic problems is the essence of mathematics and developing such ability should be the primary goal of mathematics teaching. Many would argue that a primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems. Thus, mathematics instruction should be designed so that students experience mathematics as problem solving.

Adaptive Reasoning

Adaptive reasoning is “the capacity to think logically about the relationships among concepts and situations.”⁵ Adaptive reasoning is the “glue that holds everything together, the lodestar that guides learning.”⁶ The importance of adaptive reasoning cannot be understated. Students with adaptive reasoning can think logically about the math and they can explain and justify what they are doing.

Productive Disposition

Productive disposition is “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics....Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics.”⁷

This balance of all five components is crucial to successful and effective mathematics teaching, and ultimately, to teaching for student understanding. An effective mathematics program must focus on building students’

mathematical proficiency by helping them develop these five critical components.

Teaching for Understanding in *enVision*

In planning for these new programs, the *enVision* author teams focused on implementing instructional models that would provide students with rich and varied opportunities to become mathematically proficient students. The teams looked to ensure that each component in the proficiency strand was adequately represented.

“An effective mathematics program must focus on building students’ mathematical proficiency by helping them develop these five critical components.”

In both *enVision Mathematics* Grades 6–8 and *enVision A/G/A*, each lesson opens with a problem-based activity in which students work collaboratively to draw on their existing math knowledge base to consider possible models for the problem presented, to respond to a mathematical argument presented, or draw generalizations through an exploration. In the whole-class discussion that follow the collaborative work, the probing questions provided in the Teacher’s Editions help the teacher set students up for the new learning of the lesson, presented in the next part of the lesson.

These activities target three of the competencies: strategic competence, adaptive reasoning, and productive disposition. Because these activities are student-centered activities, students need to think logically about relationships presented in the activities (adaptive reasoning), to formulate and represent mathematical problems (strategic competence), and see sense and value in mathematics (productive disposition).

Acknowledgments

⁴ National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*, Washington, DC: National Academy Press, p. 124

⁵ National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*, Washington, DC: National Academy Press, p. 129

⁶ *ibid*

⁷ National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*, Washington, DC: National Academy Press, p. 131

Understand & Apply

In part 2 of the lesson, the focus is on conceptual understanding and procedural fluency as new concepts are presented through a series of visually-rich examples. In *enVision A/G/A*, every lesson has a *Conceptual Understanding* example that focuses on making explicit the mathematical concept presented. In both programs, skill examples are designed to help students develop procedural fluency based on conceptual understanding. Each lesson ends with a formative assessment activity. The questions in the *Do You Understand?* section target students' conceptual understanding while the *Do You Know How?* exercises focus on procedural fluency.

enVision A/G/A students also have *Habits of Mind* questions and *Mathematical Practices* call out boxes. The *Habits of Mind* questions encourage students to pause and reflect on their learning, and to make connections among concepts, emphasizing logical thinking and sense making (adaptive reasoning and productive disposition).

In both programs, the *Mathematical Practices* call out boxes often include open-ended questions that help students formulate appropriate questions (strategic competence), make sense of the content they are learning (adaptive reasoning), or see patterns in the mathematics (productive disposition).

Though strategic competence and productive disposition are two areas of focus for the *enVision A/G/A* authorship team, many of the authors from both programs are former math teachers who appreciate the challenge of working with teens who often lack interest in learning math or fail to see the relevance of mathematics in their daily lives. The programs include two particular opportunities for students to perceive math as useful and worthwhile: the Mathematical Modeling in 3 Act tasks and the *enVision* STEM projects.

The *Mathematical Modeling in 3 Acts* tasks ask students to develop a mathematical model that can explain a real-world phenomenon and to use that model to answer the main question posed. These high-interest, engaging activities also help develop students' conceptual understanding, procedural fluency, and adaptive reasoning as they test out different models and conjectures to answer the question posed.

The *enVision* STEM projects are also high-interest, engaging activities that are more open-ended, encouraging students to explore real-world applications of math concepts. Like the Mathematical Modeling in 3 Acts, these activities help students develop all of the competencies that make up mathematical proficiency.

enVision sets as a program goal to provide multiple opportunities for students to develop these essential competencies and become mathematically proficient students.

Mathematics teaching in the United States too often produces students more proficient in procedural fluency and less proficient in the other four strands. To achieve the goal of student understanding of mathematics, then all five strands—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition—must receive attention in our classrooms. *enVision Mathematics* Grades 6-8 and *enVision A/G/A* provide multiple opportunities for students to develop these essential competencies and become mathematically proficient students.

References

National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

NAEP (2003). "What Does the NAEP Mathematics Assessment Measure?" Online at <http://www.nces.ed.gov/nationsreportcard/mathematics/abilities.asp>.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

SAVVAS
LEARNING COMPANY

Savvas.com
800-848-9500

Copyright © 2021 Savvas Learning Company LLC. All Rights Reserved.
Savvas™ and Savvas Learning Company™ are the exclusive trademarks of Savvas Learning Company LLC in the US and in other countries.

Join the Conversation
@SavvasLearning



Get Fresh Ideas for Teaching
[Blog.Savvas.com](https://www.blog.savvas.com)