SAVVAS

Grade 4

Let's Investigate Take and Teach

enVision+ MATHEMATICS

Real-World Math. Real-World Ready.

Let's Investigate

Lesson

8-2 Joining Equal Fractions

FOCUS

Mathematics Objective Add equal fractions using prior knowledge of fraction addition and multiplication and various pictorial and concrete representations.

Language Objective Describe how to combine equal fractions and use writing to compare repeated addition and multiplication of fractions.

Essential Understanding The product of a fraction and a whole number can be interpreted as repeated addition.

COHERENCE

Look Back Previously, students used fraction strips or number lines to understand a fraction as a multiple of a unit fraction.

This Lesson Students use different strategies and models to combine equal fractions. Students show that repeated addition of equal fractions is the same as multiplying the fraction and a whole number.

Look Ahead The next lesson explicitly develops the strategy of using models to multiply a fraction by a whole number.

Cross-Cluster Connection Building fractions from unit fractions connects to solving word problems involving multiplicative comparisons.

RIGOR

Conceptual Understanding Students use any strategy to combine equal fractions. Students show that adding equal fractions is the same as multiplying a fraction and a whole number. They write examples of repeated addition as multiplication expressions.

Vocabulary

none

Materials

- Fraction models: strips (or Teaching Tool 27)

Student Resources

Family Engagement

Intervention System (IS)
H47 Multiplying Fractions by Whole Numbers

Teacher Resources

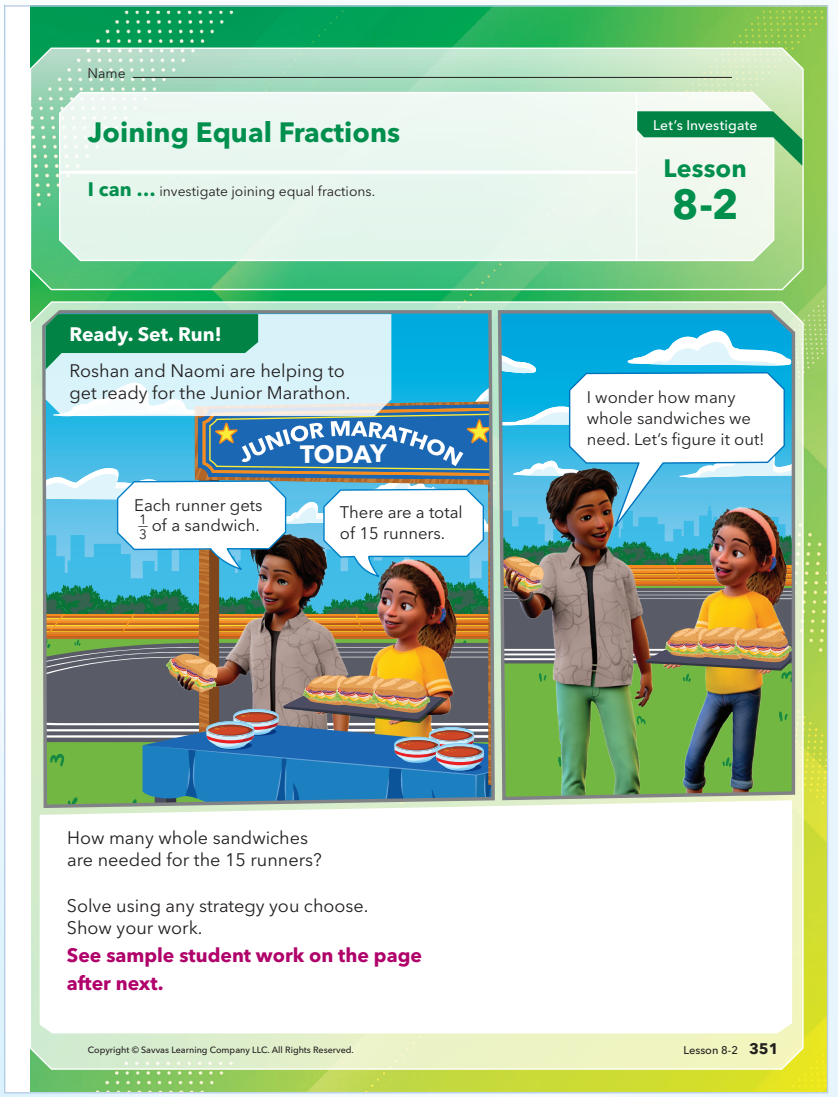
enVision on the Go

Exit Ticket Support for using the Exit Ticket is available online. Use the Exit Ticket results to support students as needed by reminding them that they can use multiplication instead of repeated addition of fractions.

Step 1 Investigate

10-25 min

Investigate Scout Observational Assessment



Name _____

Joining Equal Fractions

I can ... investigate joining equal fractions.

Let's Investigate

Lesson 8-2

Ready. Set. Run!

Roshan and Naomi are helping to get ready for the Junior Marathon.

How many whole sandwiches are needed for the 15 runners?

Solve using any strategy you choose. Show your work.

See sample student work on the page after next.

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How many cups of juice does each runner drink in all at the 6 stations?

Solve using any strategy you choose. Show your work.

See sample student work on the next page.

Build G.R.I.T. Develop a growth mindset.

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MTP Anticipate

Anticipate students' solution strategies. Think of various ways students might use their prior knowledge to solve the problems. See the next page for sample student work.

Introduce the Investigation

Use the Slow Reveal Slides You might use the slides to reveal part of the information. Then ask: What do you notice? What do you wonder?

Elevate Student Voice: Engage Students in the Context Have students bring their own unique experiences and ideas to the situation by using prompts like these, ask:

- Tell about a time you competed in a race or a sporting event.
- Have you ever shared a sandwich? How did you divide the sandwich into parts? How many parts were there?

Check for Understanding

MLR6 Three Reads

Read the task to the class. Ask:

- What is the task about?
- What do Roshan and Naomi need to prepare for the race?

Have students read the task by themselves. Ask:

- What information is given in the task?
- What did Roshan notice about the drinks?

Have a student read the task to the class. Ask:

- What is the task asking you to find?
- What questions did Roshan and Naomi ask?

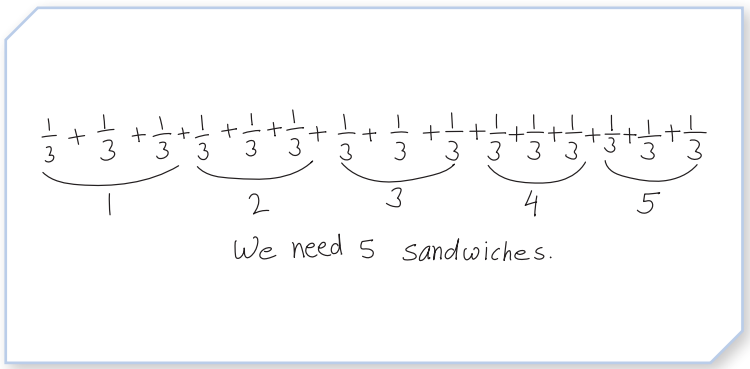
Step 1 Investigate *continued*

Sample Student Work

Ready. Set. Run! Problem A

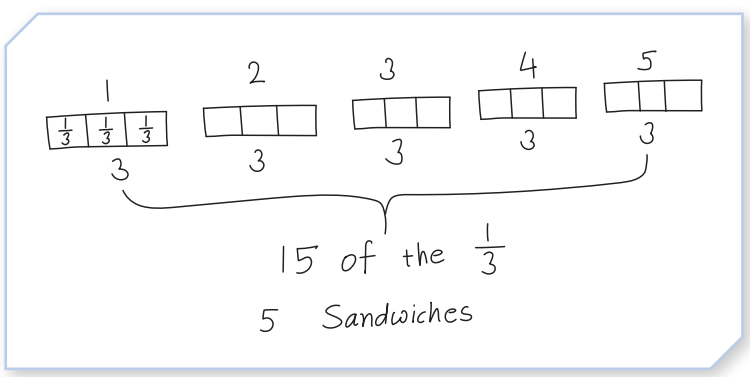
In a race, each runner gets $\frac{1}{3}$ of a sandwich. There are a total of 15 runners in the race. Use any strategy to find how many whole sandwiches are needed for the runners (join equal fractions).

- **Jamal added $\frac{1}{3}$ 15 times.** Then he combined the fractions that added to one whole. He counted the number of "one wholes" and found the total number of sandwiches.



You might ask Jamal: Is there a multiplication fact you could have written for $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$? [Listen for "$\frac{1}{3} \times 3 = 1$ or $3 \times \frac{1}{3} = 1$."]

- **Amirah used a model to show how to break 1 sandwich into 3 parts.** She then repeated her model until she had 15 parts for the 15 runners. Then she added to find the total number of sandwiches needed.

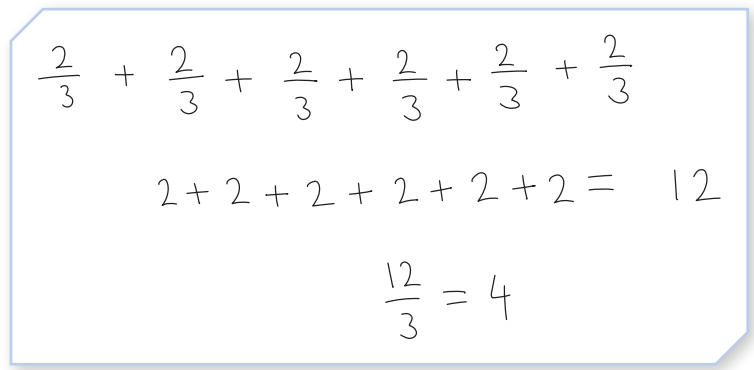


You might ask Amirah: How would the calculations change if you split the sandwich into 5 parts instead of 3? [Listen for "Each runner would receive $\frac{1}{5}$ sandwich instead of $\frac{1}{3}$. Only 3 whole sandwiches would be needed."]

Ready. Set. Run! Problem B

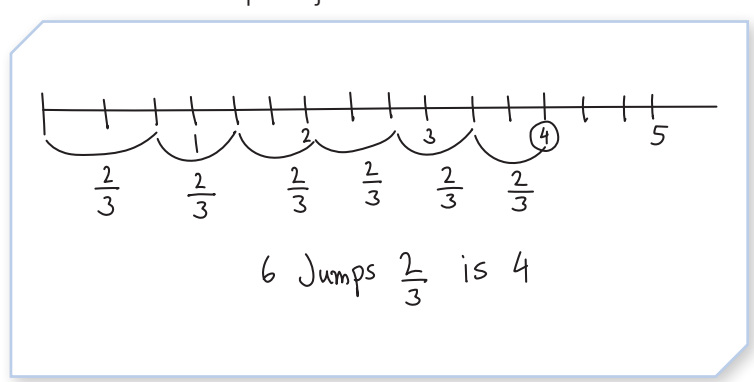
In a race, there are 6 drink stations. Each runner drinks $\frac{2}{3}$ cup of juice at each station. Use any strategy to find how much juice each runner drinks in all (join equal fractions).

- **Alice added $\frac{2}{3}$ cup 6 times.** She added the numerators and got 12. Then she divided by the denominator which was 3. She found that each runner will drink 4 cups of juice.



You might ask Alice: What does 2 + 2 + 2 + 2 + 2 + 2 show? [Listen for "Each 2 is the numerator of the fraction of juice the runner drank at each station. It shows how many parts of 1 cup of juice that the runners drank in each station."]

- **Miguel used a number line to show 6 jumps of $\frac{2}{3}$.** He started at zero and each jump moved $\frac{2}{3}$ places on the number line. He used 6 jumps for the 6 drink stations. He saw that 6 jumps of $\frac{2}{3}$ landed on the number 4, so the runners drank 4 cups of juice in total.



You might ask Miguel: Why did you make each jump $\frac{2}{3}$? [Listen for "That is the amount of juice in each cup, so it is the fraction of juice at each station."]

Step 1 Investigate *continued*

MTP Monitor

Monitor Students at Work Ask questions to assess and advance students' thinking.

Assessing Questions

Questions for Students to Understand Their Thinking

- Ask: Can you tell/show me what you did?
- Ask: How does what you wrote relate to combining equal fractions?

Questions for You as You Observe

- What strategies are students using to solve the problem?
- What prior knowledge are they using to help solve the problems?

Advancing Questions

Questions to Ask Students Who Need Help

- If students cannot get started, ask: How many equal parts are you joining together?
- If students' work is incorrect, ask: How did you get that answer? Listen for understandings and build on what they know.

Questions to Ask Students to Think More Deeply

- Ask: Is there another way to solve the problems?
- Ask: What is an equation you could write or model you could use to solve the problems?

Early Finishers

Ask: How would you find the number of whole sandwiches that would be needed for 21 runners? [Sample answer: I would add $\frac{1}{3}$ 21 times instead of 15 times. There would be 7 whole sandwiches needed if each runner had $\frac{1}{3}$ of a sandwich.]

MTP Select and Sequence

Select and Sequence Students' Solutions for them to present with the goal of building understanding of key concepts.

Start with solutions that show repeated addition of fractions. (Jamal)

Then show solutions that only add the numerator and keep the common denominator. (Alice)

Finally, show solutions that use a model, such as a drawing or number line, to visualize and solve the problem. (Amirah and Miguel)

Step 2 Connect

10-55 min

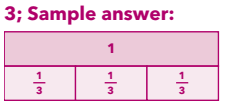
MTP Connect

Name _____

Connect

1. **Make Sense and Persevere** Why were 15 whole sandwiches not needed for the 15 runners? Explain.
   Sample answer: Each runner gets $\frac{1}{3}$ sandwich which is less than 1 whole sandwich. When I add up 15 groups of $\frac{1}{3}$ that's less than 15 groups of 1.

2. **Model with Math** How many $\frac{1}{3}$s make one whole? Draw a picture to show how you know.
   3; Sample answer:



3. **Use Structure** Is the sum of $\frac{3}{4} + \frac{3}{4}$ the same as the product of $2 \times \frac{3}{4}$? Explain.
   Yes; Sample answer: $2 \times \frac{3}{4}$ means 2 groups of $\frac{3}{4}$.

Talk About Ideas What operation(s) can you use to join equal fractions?

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Connect Solution Strategies Discuss students' work. Ask how the solutions are similar and different. Help students see that Jamal and Alice used repeated addition to combine the equal fractions. Amirah and Miguel used a model or a number line to see the parts of the whole.

Connect to Key Concepts Use the Connect questions to build understanding of the key concept at the top of the next page.

MTP Talk About Ideas

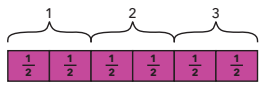
Have students engage in a think-pair-share to discuss the strategies. Listen for students who prefer to use a model so they can visualize the parts of the whole. Others might prefer to add the fractions using a familiar operation.

Step 3 Practice and Problem Solving

10-15 min

Interactive Practice Adaptive Practice

If 6 students each get $\frac{1}{2}$ of a sandwich, how many sandwiches are needed?



6 groups of $\frac{1}{2}$ is 3 wholes.

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=3$

$6\times\frac{1}{2}=3$

When joining equal fractions, you can use repeated addition to find the total. Adding equal fractions is the same as multiplying a whole number by a fraction.

Practice and Problem Solving

Lesson Help

For **4–7**, find the total. Then write the repeated addition as a multiplication expression.

4. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$ $1\frac{1}{3}$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ is the same as

4 × $\frac{1}{3}$.

5. $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=$ $1\frac{1}{2}$

$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$ is the same as

3 × $\frac{1}{2}$.

6. $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=$ 2

$\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$ is the same as

5 × $\frac{2}{5}$.

7. $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=$ $3\frac{3}{4}$

$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$ is the same as

5 × $\frac{3}{4}$.

8. Mariana is making 5 recipes of trail mix.
How many cups of raisins does she need?
How many cups of peanuts does she need?
$1\frac{1}{4}$ cups raisins; $3\frac{1}{3}$ cups peanuts

Trail Mix Recipe
$\frac{1}{4}$ cup raisins
$\frac{2}{3}$ cup peanuts

Assessment Practice

9. Which is equal to 2? 4.NF.4b

Ⓐ $\frac{1}{4}+\frac{1}{4}$
Ⓑ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$
Ⓒ $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$
Ⓓ $\frac{3}{4}+\frac{3}{4}$

10. Which is **NOT** equal to $7\times\frac{1}{5}$? 4.NF.4a

Ⓐ $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$
Ⓑ $\frac{5}{7}$
Ⓒ $\frac{7}{5}$
Ⓓ $1\frac{2}{5}$

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MLL Multilingual Learners Reading

Tell students that a Junior Marathon is a running race. The runners run through the course as fast as they can. The runners stop to drink juice along the way. They eat part of a sandwich at the end of the race.

Emerging Reread the problem with students. Identify the information students will need to solve the problem. Have students underline or circle the information they will need from the problem.

Expanding Have students read the problem aloud. Ask students how they could use the operation of addition to solve this problem, and what numbers they could add to find the answer. Encourage students to use a model if needed.

Bridging Have students read the problem with a partner, and discuss the strategies and steps they will use to solve the problem. Encourage students to think of more than one strategy they could use.

Reflection

Have students compare using repeated addition of equal fractions and multiplication of a whole number by a fraction. Ask: Do you arrive at the same answer when using the different operations or strategies? [Yes. I got an answer of 5 using both strategies.] Say: Explain how you used each operation to find the answer. [When I count $\frac{1}{3}$ 15 times, I get $\frac{15}{3}$ which is equal to 5. When I multiply $\frac{1}{3}$ by 15, I also get $\frac{15}{3}$, which is equal to 5.] Ask: Why are you able to use both addition and multiplication to find the answer? [Adding equal fractions is the same as multiplying a whole number by a fraction.]

Error Intervention Item 4
If students do not currently add the numerators and keep the common denominator,
then remind them that when adding fractions, we add only the numerator and keep the common denominator in the answer.

Item 8 If students are struggling to add the fractions, encourage them to use fraction strips or another model to identify the answer.

MTP Quick Check

Formative Assessment

Check marks indicate items for prescribing differentiation. Item 9: 1 point. Items 4 and 8: up to 2 points. The online Quick Check is auto scored and can auto prescribe differentiation.

Step 4 Assess and Differentiate

10-15 min

Additional Practice

Name _____

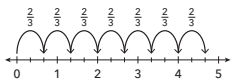
Additional Practice

Joining Equal Fractions

8-2

Another Look!

7 groups of $\frac{2}{3}$ is $4\frac{2}{3}$.



$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\frac{14}{3}=4\frac{2}{3}$

$7\times\frac{2}{3}=4\frac{2}{3}$

You can use repeated addition or multiplication to join equal fractions.

Video

For **1–5**, find the total. Then write the repeated addition as a multiplication expression.

1. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=$ 1

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ is the same as

4 × $\frac{1}{4}$.

2. $\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}=$ $2\frac{2}{8}$

$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$ is the same as

6 × $\frac{3}{8}$.

3. $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=$ $2\frac{1}{4}$

$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$ is the same as

3 × $\frac{3}{4}$.

4. $\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}=$ 4

$\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}+\frac{4}{5}$ is the same as

5 × $\frac{4}{5}$.

5. If you join 10 groups of $\frac{2}{3}$, what do you get? Write both an addition equation and a multiplication equation to represent the situation.

$6\frac{2}{3}$; $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=6\frac{2}{3}$;

$10\times\frac{2}{3}=6\frac{2}{3}$

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6. Model With Math Kayla rides her bike for $\frac{3}{4}$ hour each week. After 6 weeks, how many hours will she have ridden her bike? Write both an addition equation and a multiplication equation to represent the situation.

$4\frac{1}{2}$ hours;

$\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=4\frac{2}{4}$;

$6\times\frac{3}{4}=4\frac{2}{4}$

7. Generalize Sunita is using repeated addition to add equal unit fractions. She notices a pattern.

$\frac{1}{2}+\frac{1}{2}=1$

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$

Describe the pattern and use it to write the next line in the pattern. What is the sum if you add 8 groups of $\frac{1}{8}$?

Sample answer: If you're adding groups of equal unit fractions, and the number of groups is the same as the denominator, the sum is 1. The next line is $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=1$. 8 groups of $\frac{1}{8}$ is equal to 1.

8. Higher Order Thinking Omar mixes $\frac{2}{3}$ cup of sparkling water with $\frac{1}{4}$ cup of juice to make one serving of fruit punch. If he mixes 12 servings in all, will the punch fit in the pitcher? Explain.

No; Sample answer: In 1 serving, there is $\frac{8}{12}+\frac{3}{12}=\frac{11}{12}$ cup of fruit punch. For 12 servings, add 12 groups of $\frac{11}{12}$ cups, which is $\frac{132}{12}=11$ cups. 11 cups will not fit in the pitcher.

Assessment Practice

9. Which is equal to $2\frac{2}{5}$?

Ⓐ $\frac{2}{5}+\frac{2}{5}$

Ⓑ $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$

Ⓒ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

Ⓓ $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$

10. Which is **NOT** equal to $8\times\frac{1}{3}$?

Ⓐ $2\frac{2}{3}$

Ⓑ $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

Ⓒ $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

Ⓓ $\frac{8}{3}$

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Differentiation Library

The **Differentiation Library** at the beginning of this topic shows the differentiation options available, each of which can be appropriate for all students. However, based on the **Quick Check** results, you may choose to prescribe:

0–3 points: Intervention Activity or Reteach to Build Understanding Activity

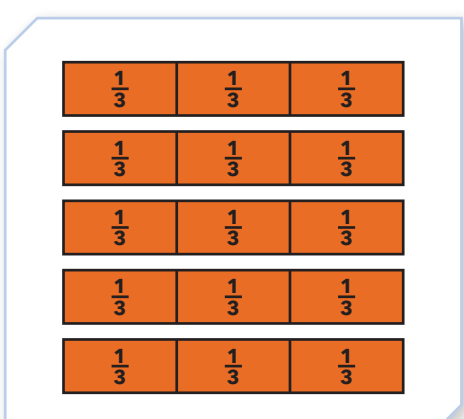
4–5 points: Enrichment Activity

In addition, **multilingual learners** may benefit from the Build Mathematical Literacy Activity.

Intervention Activity

Materials Fraction models: strips (or Teaching Tool 27)

- Have students use the fraction strips to represent the part of the sandwich that each runner will get. Each fraction strip represents $\frac{1}{3}$ of a sandwich. Have students group 3 strips together to show one whole sandwich. Then have students repeat this process until they use 15 strips, one for each runner. Students should count their groups to find that they need 5 whole sandwiches for 15 runners.
- Guide students to write an addition equation that relates to their model. Students should add $\frac{1}{3}$ the total of 15 times. Then show students that they could also multiply 15 by $\frac{1}{3}$ to find the same answer. Encourage students to use repeated addition and multiplication as strategies to solve the problem.



- Have students work in pairs and repeat the process for the second problem. Encourage students to use both addition and multiplication strategies to find the answer.

Video Tutorial

Another Look Video

Intentional Blank Page

Name _____

Joining Equal Fractions

Let's Investigate

Lesson 8-2

I can ... investigate joining equal fractions.

Ready. Set. Run!

Roshan and Naomi are helping to get ready for the Junior Marathon.

How many whole sandwiches are needed for the 15 runners?

Solve using any strategy you choose. Show your work.

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How many cups of juice does each runner drink in all at the 6 stations?

Solve using any strategy you choose. Show your work.

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Name _____

Connect

1. **Make Sense and Persevere** Why were 15 whole sandwiches not needed for the 15 runners? Explain.

2. **Model with Math** How many $\frac{1}{3}$s make one whole? Draw a picture to show how you know.

3. **Use Structure** Is the sum of $\frac{3}{4} + \frac{3}{4}$ the same as the product of $2 \times \frac{3}{4}$? Explain.

Talk About Ideas What operation(s) can you use to join equal fractions?

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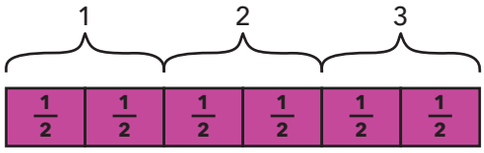
If 6 students each get $\frac{1}{2}$ of a sandwich, how many sandwiches are needed?



6 groups of $\frac{1}{2}$ is 3 wholes.

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3$

$6 \times \frac{1}{2} = 3$

When joining equal fractions, you can use repeated addition to find the total. Adding equal fractions is the same as multiplying a whole number by a fraction.

Practice and Problem Solving

Lesson Help

For **4–7**, find the total. Then write the repeated addition as a multiplication expression.

4. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$ _____

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ is the same as

_____ × _____.

5. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$ _____

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ is the same as

_____ × _____.

6. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} =$ _____

$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$ is the same as

_____ × _____.

7. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} =$ _____

$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$ is the same as

_____ × _____.

8. Mariana is making 5 recipes of trail mix.
How many cups of raisins does she need?
How many cups of peanuts does she need?

Trail Mix Recipe

$\frac{1}{4}$ cup raisins

$\frac{2}{3}$ cup peanuts

Assessment Practice

9. Which is equal to 2?

Ⓐ $\frac{1}{4} + \frac{1}{4}$

Ⓑ $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

Ⓒ $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

Ⓓ $\frac{3}{4} + \frac{3}{4}$

10. Which is **NOT** equal to $7 \times \frac{1}{5}$? 4.NF.4a

Ⓐ $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

Ⓑ $\frac{5}{7}$

Ⓒ $\frac{7}{5}$

Ⓓ $1\frac{2}{5}$

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