SAVVAS

Grade 8

Let's Investigate Take and Teach

enVision+ MATHEMATICS

Real-World Math. Real-World Ready.

Let's Investigate

Lesson

2-2

Solve Equations with Variables on Both Sides

FOCUS

Mathematics Objectives

- Solve equations with like terms on both sides of the equation.
- Make sense of scenarios and represent them with equations.

Language Objective Explain, orally and in writing, how to solve equations with variables on both sides of the equal sign.

Essential Understanding To solve a linear equation that has variable terms on both sides of the equation, first use inverse operations to move all variable terms to one side of the equation and constant terms to the other. Then, combine like terms and use inverse operations to isolate the variable.

COHERENCE

Previously in this topic, students:

- combined like terms in an equation with variable terms on only one side of the equation.
- solved one- and two-step equations.

In this lesson, students:

- combine like terms from both sides of an equation onto one side by using inverse operations.
- solve equations to find the value of a variable.

Later in this topic, students will:

- solve multistep equations with like terms on both sides of the equation.

Cross-Cluster Connection Solving equations connects to understanding connections between proportional relationships, lines, and linear functions later in this topic.

RIGOR

Conceptual Understanding Students will become fluent in solving equations with variables on both sides.

Application Students solve real-world problems by combining like terms and using inverse operations to solve for the variable.

Vocabulary

none

Materials

- pan balance
- algebra tiles

Student Resources

Family Engagement

Intervention System (IS)

K26 Solving Equations with Decimals
K29 Solving Equations with Fractions

Teacher Resources

Envision on the Go

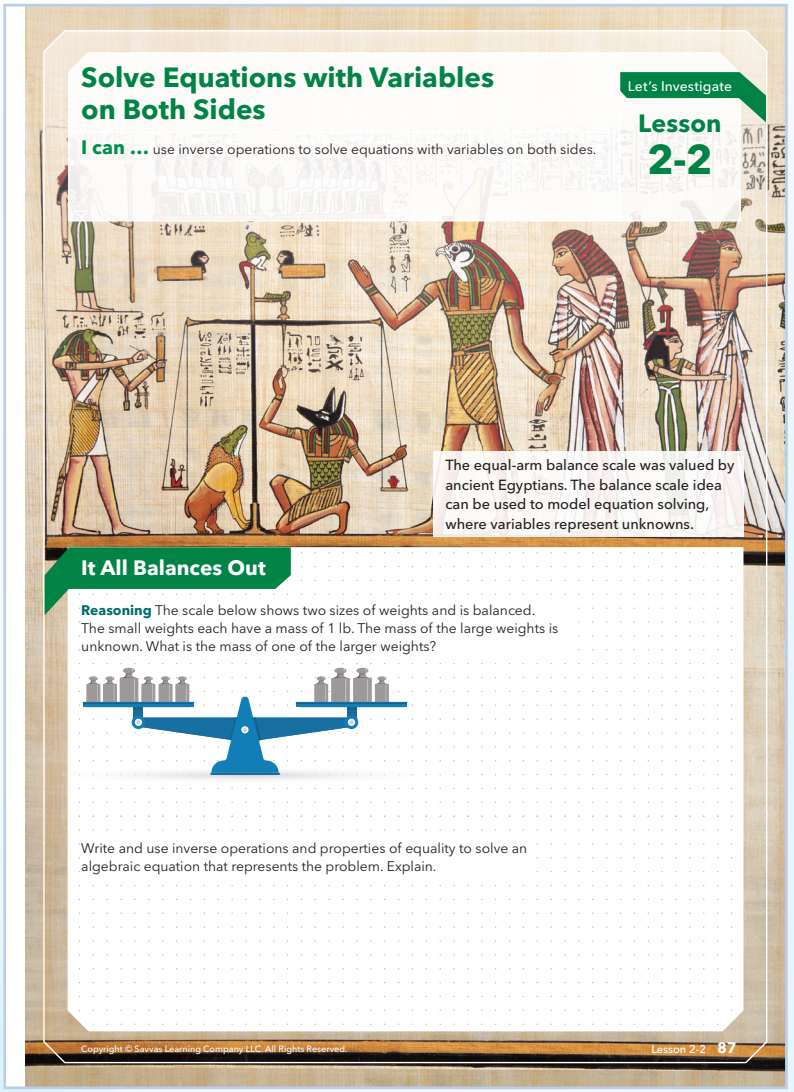
Exit Ticket

Support for using the Exit Ticket is available online.

Step 1 Investigate

30–35 min

Investigate Scout Observational Assessment



Solve Equations with Variables on Both Sides

Let's Investigate

Lesson 2-2

I can ... use inverse operations to solve equations with variables on both sides.

The equal-arm balance scale was valued by ancient Egyptians. The balance scale idea can be used to model equation solving, where variables represent unknowns.

It All Balances Out

Reasoning The scale below shows two sizes of weights and is balanced. The small weights each have a mass of 1 lb. The mass of the large weights is unknown. What is the mass of one of the larger weights?

Write and use inverse operations and properties of equality to solve an algebraic equation that represents the problem. Explain.

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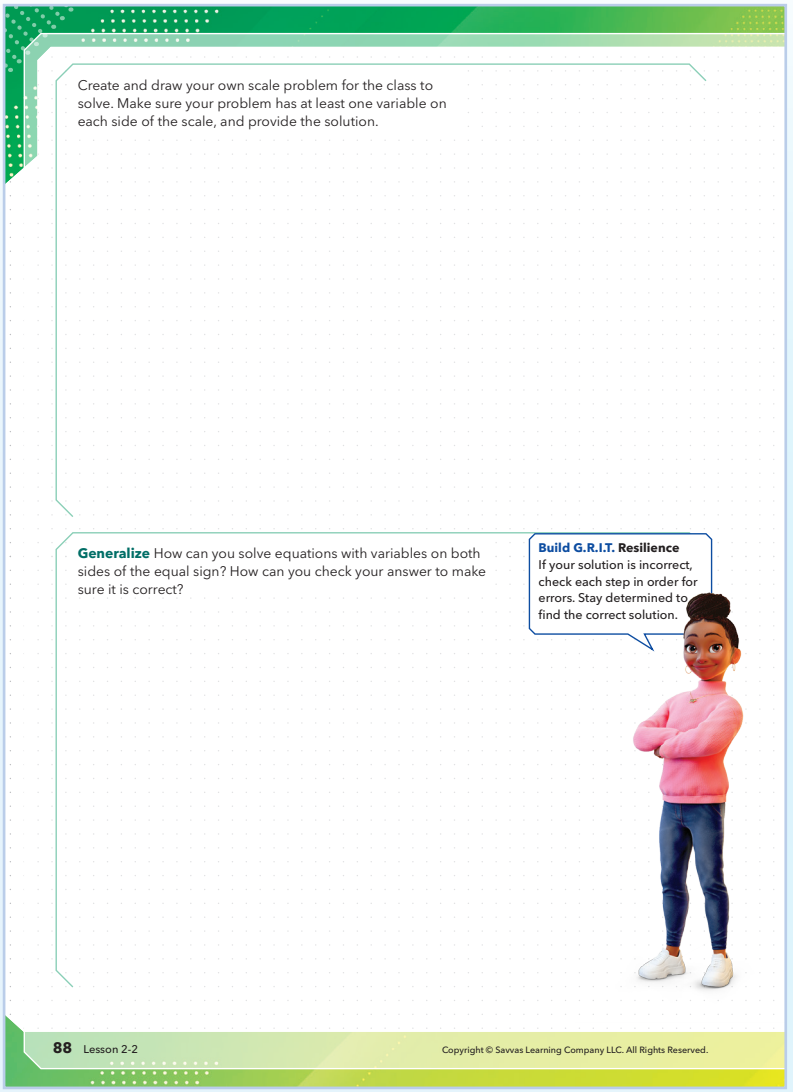

Create and draw your own scale problem for the class to solve. Make sure your problem has at least one variable on each side of the scale, and provide the solution.

Generalize How can you solve equations with variables on both sides of the equal sign? How can you check your answer to make sure it is correct?

Build G.R.I.T. Resilience
If your solution is incorrect, check each step in order for errors. Stay determined to find the correct solution.

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MTP Anticipate

Anticipate students' solution strategies. Think of various ways students might solve simple equations with variables on both sides. See the next page for sample student work.

Introduce the Investigation

Elevate Student Voice: Engage Students in the Context

- What is something from your experience that has balance?
- What do you know about ancient Egypt?
- What might ancient Egyptians have needed to weigh?

Use the Slow Reveal Slides

Reveal part of the information. Ask: What do you notice? What do you wonder?

Check for Understanding

MLR8 Discussion Supports

Focus on the introductory context of the problem. For each sentence, have a student read the sentence aloud. Have another student revoice the sentence as you write it on the board to display. Samples:

- The mass is equal on both sides of the scale.
- The smaller weights have a mass of 1 pound each.
- We don't know the mass of the larger weights.
- We have to find the mass of a large weight.

Step 1 Investigate *continued*

Sample Student Work

Student 1 Work

- **correctly finds the mass using guess and check**

$2 + 4 + 3 = 1 + 4 + 4 + 1$
$9 = 10$ X

$2 + 3 + 3 = 1 + 3 + 3 + 1$
$8 = 8$

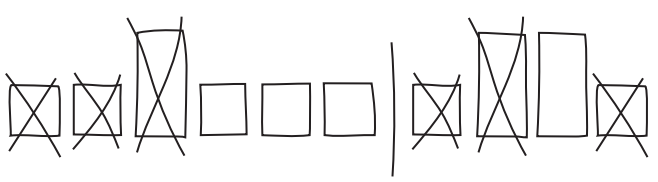
The mass of the weight is 3 because it makesthe scale balance. When the mass is 3, each side of the scale has a mass of 8.

$1 + 1 + x + 1 + 1 + 1 = 1 + x + x + 1$
$5 + x = 2 + 2x$
$5 = 2 + 2x - x$
$5 - 2 = 2 + x - 2$
$3 = x$

Ask: Which weight is represented by a variable when you write it as an equation? [Since a variable represents an unknown, the large weight is represented by a variable.]

Student 2 Work

- **correctly creates model and equation using properties of equality**



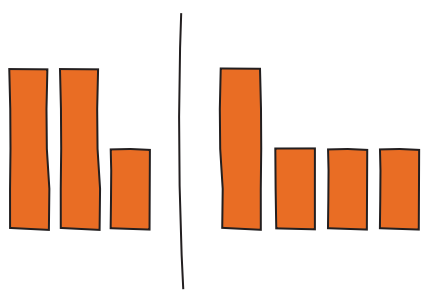
$3 = x$

$x + 5 = 2x + 2$
$5 = x + 2$
$3 = x$
The mass of the larger weight is 3.

Ask: Why are there variables on both sides of the equation? [There is at least one unknown mass on each side of the scale.]

Student 3 Work

- **incorrectly models and solves an equation**

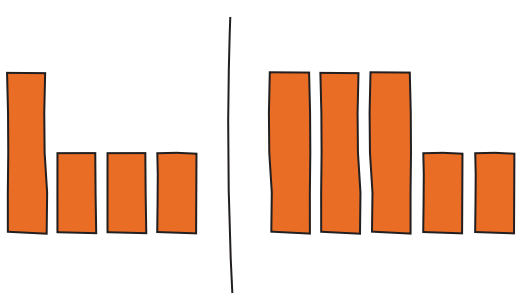


$2x + 1 = x + 3$
$x + 1 = 3$
$x = 2$

Ask: Does everyone agree with that solution? [Allow any student who disagrees to explain what they think is wrong and show what they think is the correct solution.]

Student 4 Work

- **incorrectly uses multiplication while solving**



$x + 3 = 3x + 2$
$1 = 2x$
$x = 2$

Ask: Does everyone agree with that solution? [Allow any student who disagrees to explain what they think is wrong and show what they think is the correct solution.]

MTP Monitor

Monitor Students at Work Ask questions to assess and advance students' thinking.

Assessing Questions

Questions for Students to Understand Their Thinking

- Does the scale shown look like a balance you are familiar with?
- How would you describe what is happening with the scale in It All Balances Out? ?

Questions for You as You Observe

- Which mass represents a variable?
- How many of the variable is there on each side of the equation?
- How do you get all the variables on one side of the equation?

Advancing Questions

Questions for Students Who Need Help

- If you take a weight away from one side, how do you get the scale to balance again?
- Which mass represents a constant number?
- What does the equal sign represent?
- What does each large mass represent?
- How can you turn your scale into an equation?

Questions to Help Students Think More Deeply

- What happens if you start with the same number of masses that represent a variable on both sides of the equation?

MTP Select and Sequence

Select and Sequence Students' Solutions for presentations with the goal of building understanding of key math concepts.

- **Start** with correct solutions that use a strategy other than writing and solving equations (Student 1 and Student 2 Work).
- **Then** show a correct solution that uses inverse operations and properties of equality to solve the algebraic equation (Student 3 Work).
- **Finally,** show an incorrect solution that solves using multiplication instead of division (Student 4 Work).

MTP Discuss Student Solutions

After allowing students to finish the questions for the lesson, return to an all-class discussion.

- What is the difference between a variable and a constant? [A constant represents a known value and a variable represents an unknown value.]
- What property of equality is represented by adding the same weight to each side of an equation? [Addition Property of Equality]
- Do we have enough information to solve equations with variables on both sides? Students are likely to say yes. If they do, prompt further by asking:
- How do you know you have enough information? [We have all the steps to get the variable on one side and then solve for the variable.]

MLL Multilingual Learners SPEAKING

Emerging Multilingual learners may be unfamiliar with some vocabulary in It All Balances Out. Have students relate inverse operations and the properties of equality to the balance scale.

Inverse operations are _____ operations. The inverse operation of addition is _____. [opposite; subtraction]

Removing the same weight from each side of an equation demonstrates the _____ Property of Equality. [Subtraction].

Expanding Have students reread the problem and summarize the known information and the question aloud.

Ask students to share their summaries. Students should explain why the two sides of the scale are equal to each other and how to solve for the variable.

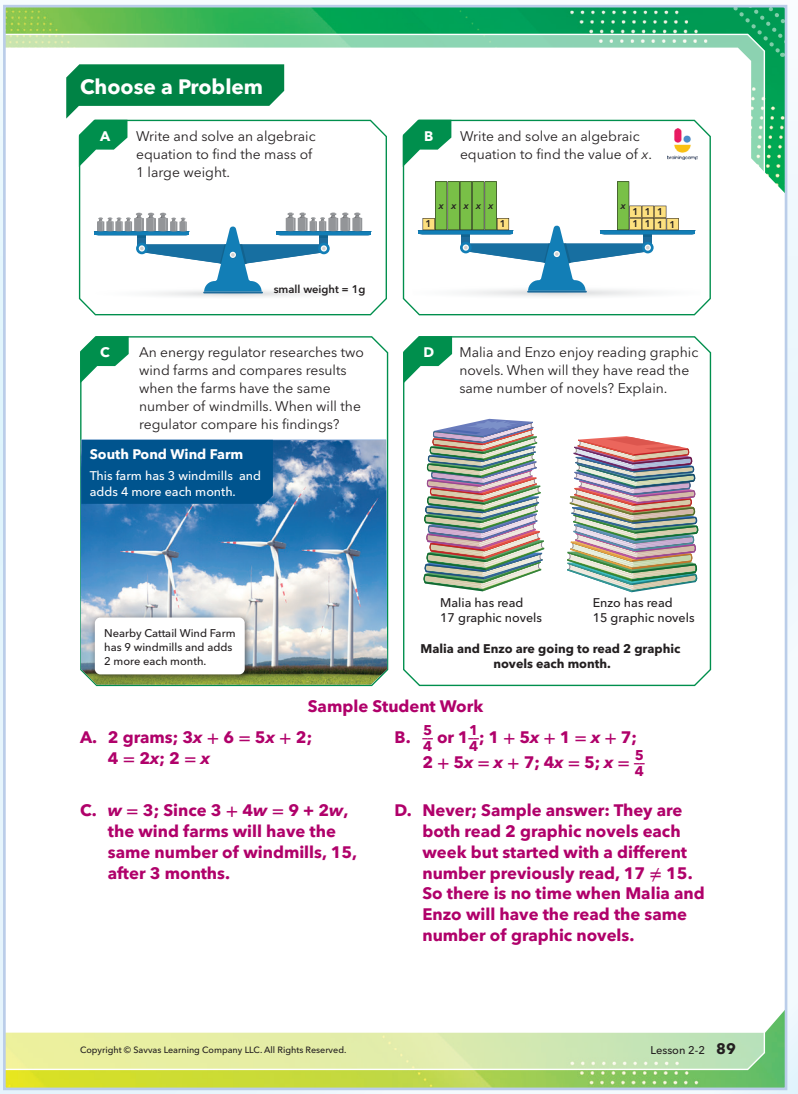
Bridging Have students reread the problem and explain the steps taken to solve for the variable.

Identify unknown vocabulary words and have students determine what they mean based on the context of the problem.

- What words were unknown and what is your definition based on the problem context?
- How did you solve for the variable?

Step 2 Connect

25–30 min



Choose a Problem

A Write and solve an algebraic equation to find the mass of 1 large weight.

small weight = 1g

B Write and solve an algebraic equation to find the value of x.

C An energy regulator researches two wind farms and compares results when the farms have the same number of windmills. When will the regulator compare his findings?

South Pond Wind Farm
This farm has 3 windmills and adds 4 more each month.

Nearby Cattail Wind Farm has 9 windmills and adds 2 more each month.

D Malia and Enzo enjoy reading graphic novels. When will they have read the same number of novels? Explain.

Malia has read 17 graphic novels

Enzo has read 15 graphic novels

Malia and Enzo are going to read 2 graphic novels each month.

Sample Student Work

A. 2 grams; $3x + 6 = 5x + 2$; $4 = 2x$; $2 = x$

B. $\frac{5}{4}$ or $1\frac{1}{4}$; $1 + 5x + 1 = x + 7$; $2 + 5x = x + 7$; $4x = 5$; $x = \frac{5}{4}$

C. $w = 3$; Since $3 + 4w = 9 + 2w$, the wind farms will have the same number of windmills, 15, after 3 months.

D. Never; Sample answer: They are both read 2 graphic novels each week but started with a different number previously read, $17 \neq 15$. So there is no time when Malia and Enzo will have the read the same number of graphic novels.

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Engage Students Through Choice

Allow students to select a problem to solve individually.

Problem A supports students who are still developing their understanding of combining like terms from both sides of an equation by using inverse operations.

Problem B introduces the idea of solving algebraic equations to find the value of a variable.

Problem C asks students to write and solve an algebraic equation without a visual representation of the problem.

Problem D allows students to use critical reasoning skills to reason about problems with no solutions.

Early finishers may choose more than one problem.

MTP Choose a Problem

Discuss Solution Strategies

Group students based on the chosen problem to discuss their solution strategies. Consider having groups record their solutions on a vertical surface. Do a gallery walk so they can discuss different problems and solution strategies.

Problem A

- What does the position of the balance tell you about the total mass on each side?

Problem B

- Will your equation have variables on one or both sides? How about constants?

Problem C

- Why would students research wind power?
- Why is it a clean, reliable source of energy?

Problem D

- What is the opposite of a problem with no solutions?

Present and Analyze

Prioritize discussion about the variable used in Problem B. If needed to promote discussion, ask questions such as:

Problem A

- How can you represent the mass of 1 large weight in your equation? [I can use the variable x to represent the mass of 1 large weight.]

Problem B

- What does x represent? [x is the unknown value in the algebraic equation.]

Problem C

- In the algebraic equation you wrote to solve this problem, what does that variable represent? [The number of months before the two farms will have the same number of windmills]

Problem D

- If you put the number of Malia's and Enzo's graphic novels on a balance scale, how would they compare? [The numbers would not be balanced because 15 is not equal to 17.]

Connect

1. **Make Sense and Persevere** How was the strategy you used in *Choose a Problem* the same as your strategy in *It All Balances Out*? How were your strategies different?
   Sample answer: You use inverse operations and properties of equality to solve the algebraic equation that represented each problem.
   A. The large weights on the right side are not grouped together.
   B. The problem is modeled with algebra tiles instead of weights.
   C. You need to write an equation from a verbal description instead of a model.
   D. There is no solution to the equation that models this problem.
2. **Use Structure** How can you turn a balance scale problem into an algebraic equation?
   You can represent each side with an algebraic expression. The unknown amounts are represented with a variable, such as *x*, and the known amounts are represented as a number or constant. The expression on the left side would then equal the expression on the right side.
3. **Model with Math** What are some ways to represent an algebraic equation?
   Sample answer: algebra tiles, balance scales
4. **Reasoning** What happens if the variable term is equivalent on both sides?
   There is no solution to the equation.
5. **Generalize** How is solving equations with variables on both sides different from solving with a variable on only one side?
   When solving an equation with variables on both sides, first you need to use the properties of equality to move the variables to one side of the equation.

Key Concept

You can use different representations, such as algebra tiles and balance scales, to model solving equations with variables on both sides.

$4x + 1 = 2x + 3$

$4x - 2x + 1 = 2x - 2x + 3$

$2x + 1 = 3$

$2x + 1 - 1 = 3 - 1$

$2x = 2$

$2x \div 2 = 2 \div 2$

$x = 1$

Talk About Ideas

How does adding or removing the same number of weights on each side of a balance scale connect to the properties of equality?

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MTP Connect

Connect Solution Strategies Use your students' work or the sample student work to facilitate responses phrased in students' own words. Encourage students to use the vocabulary *inverse operations, properties of equality,* and *variable*.

Item 1 Comparing and contrasting solutions builds understanding of solving equations with variables on both sides. Have students focus on combining like terms and the properties of equality.

Item 2 Balancing equations allows for discussion on the properties of equality that keep both sides of the equation equal.

Item 3 Representing algebraic equations with models can help foster understanding. A balanced seesaw is another way to represent an algebraic equation.

Item 4 Allow students to reason why there is no solution if the variable term is equivalent on both sides. If both sides of the equation have the term 5*x*, what happens when you subtract 5*x* from one side?

Item 5 The properties of equality allow the equation to remain balanced while combining like terms, like lifting the same weight from each side of a balance scale.

MTP Key Concept

- Why is it important to know the relationship between two expressions before setting up an equation? [You must be able to tell from the information in the problem that the expressions are equal. Then you can set them up in an equation.]

MTP Talk About Ideas

Listen for discussions about how to solve equations with variables on both sides.

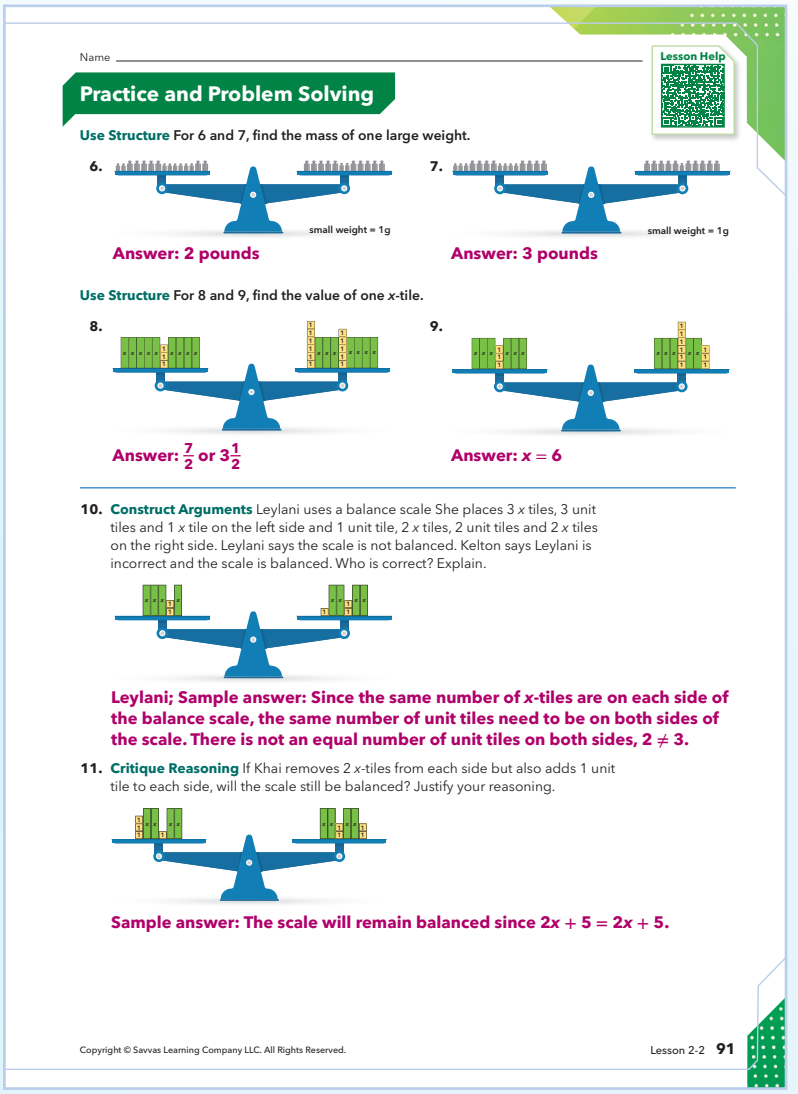
- How can you connect adding a weight to each side of the scale to the Addition Property of Equality?
- How can you connect removing a weight from each side of the scale to the Subtraction Property of Equality?

Step 3 Practice and Problem-Solving

15–20 min

Interactive Practice Adaptive Practice

You may opt to have students complete automatically scored practice items online.



Name _____

Lesson Help

Practice and Problem Solving

Use Structure For 6 and 7, find the mass of one large weight.

6. small weight = 1g

Answer: 2 pounds

7. small weight = 1g

Answer: 3 pounds

Use Structure For 8 and 9, find the value of one x-tile.

8. **Answer: $\frac{7}{2}$ or $3\frac{1}{2}$**

9. **Answer: $x = 6$**

10. **Construct Arguments** Leylani uses a balance scale She places 3 x tiles, 3 unit tiles and 1 x tile on the left side and 1 unit tile, 2 x tiles, 2 unit tiles and 2 x tiles on the right side. Leylani says the scale is not balanced. Kelton says Leylani is incorrect and the scale is balanced. Who is correct? Explain.

Leylani; Sample answer: Since the same number of x-tiles are on each side of the balance scale, the same number of unit tiles need to be on both sides of the scale. There is not an equal number of unit tiles on both sides, $2 \neq 3$.

11. **Critique Reasoning** If Khai removes 2 x-tiles from each side but also adds 1 unit tile to each side, will the scale still be balanced? Justify your reasoning.

Sample answer: The scale will remain balanced since $2x + 5 = 2x + 5$.

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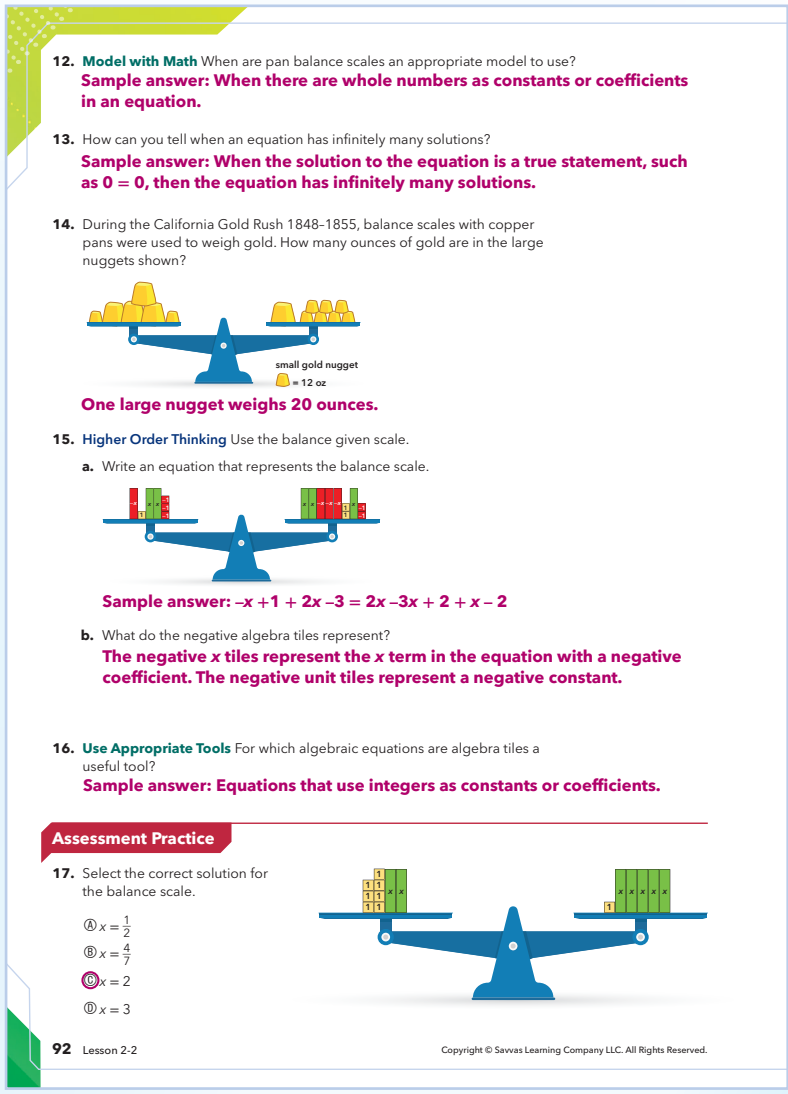

12. **Model with Math** When are pan balance scales an appropriate model to use?
Sample answer: When there are whole numbers as constants or coefficients in an equation.

13. How can you tell when an equation has infinitely many solutions?
Sample answer: When the solution to the equation is a true statement, such as $0 = 0$, then the equation has infinitely many solutions.

14. During the California Gold Rush 1848–1855, balance scales with copper pans were used to weigh gold. How many ounces of gold are in the large nuggets shown?

small gold nugget = 12 oz

One large nugget weighs 20 ounces.

15. **Higher Order Thinking** Use the balance given scale.

a. Write an equation that represents the balance scale.

Sample answer: $-x + 1 + 2x - 3 = 2x - 3x + 2 + x - 2$

b. What do the negative algebra tiles represent?
The negative x tiles represent the x term in the equation with a negative coefficient. The negative unit tiles represent a negative constant.

16. **Use Appropriate Tools** For which algebraic equations are algebra tiles a useful tool?
Sample answer: Equations that use integers as constants or coefficients.

Assessment Practice

17. Select the correct solution for the balance scale.

Ⓐ $x = \frac{1}{2}$
Ⓑ $x = \frac{4}{7}$
Ⓒ $x = 2$
Ⓓ $x = 3$

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Practice and Problem Solving

Error Intervention

Item 11 **Critique Reasoning** Some students may be confused by adding and removing tiles and may make errors due to this confusion.

- How does removing x-tiles from each side relate to the Subtraction Property of Equality? [The scale is already balanced, so removing the same number of x-tiles from each side will maintain the balance.]
- How does adding unit tiles to each side relate to the Addition Property of Equality? [The scale is already balanced, so adding the same number of unit tiles to each side will maintain the balance.]

Multilingual Learners

Item 13 What does the phrase "infinitely many solutions" mean? [Any value for the variable makes the equation true.]

Engage Through Student Choice

Go online for an assignment strategy.

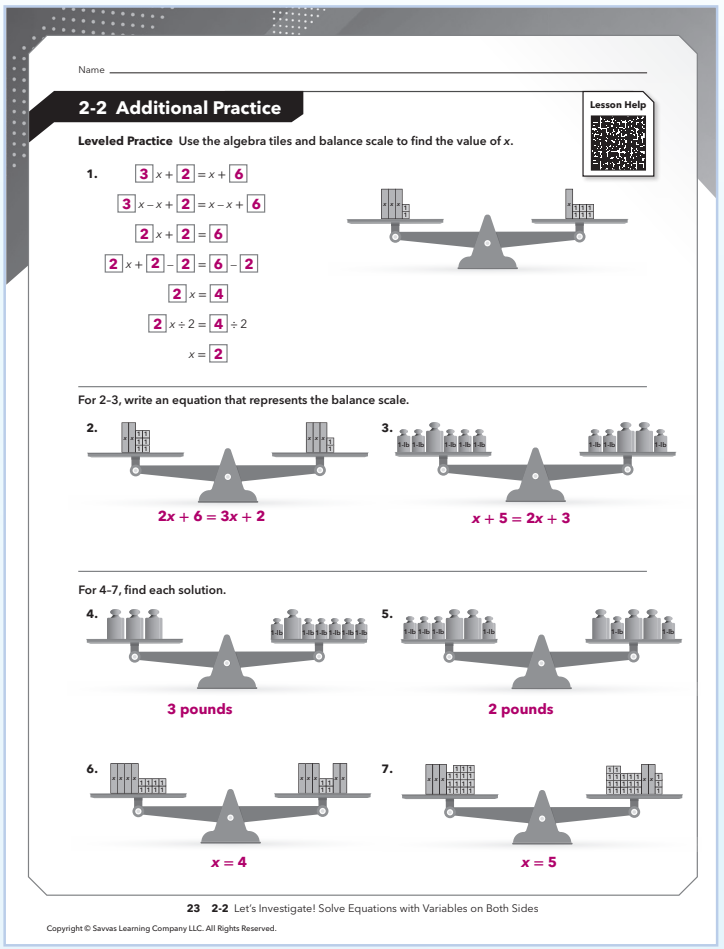
Item Analysis: Practice and Problem Solving

Items	DOK
6–9, 12, 14, 17	1
13, 16	2
10, 11, 15	3

Step 4 Assess and Differentiate

15–30 min

Additional Practice



Name _____

2-2 Additional Practice

Lesson Help

Leveled Practice Use the algebra tiles and balance scale to find the value of x.

1. $3x + 2 = x + 6$
   $3x - x + 2 = x - x + 6$
   $2x + 2 = 6$
   $2x + 2 - 2 = 6 - 2$
   $2x = 4$
   $2x \div 2 = 4 \div 2$
   $x = 2$

For 2–3, write an equation that represents the balance scale.

2. $2x + 6 = 3x + 2$
3. $x + 5 = 2x + 3$

For 4–7, find each solution.

4. 3 pounds
5. 2 pounds
6. $x = 4$
7. $x = 5$

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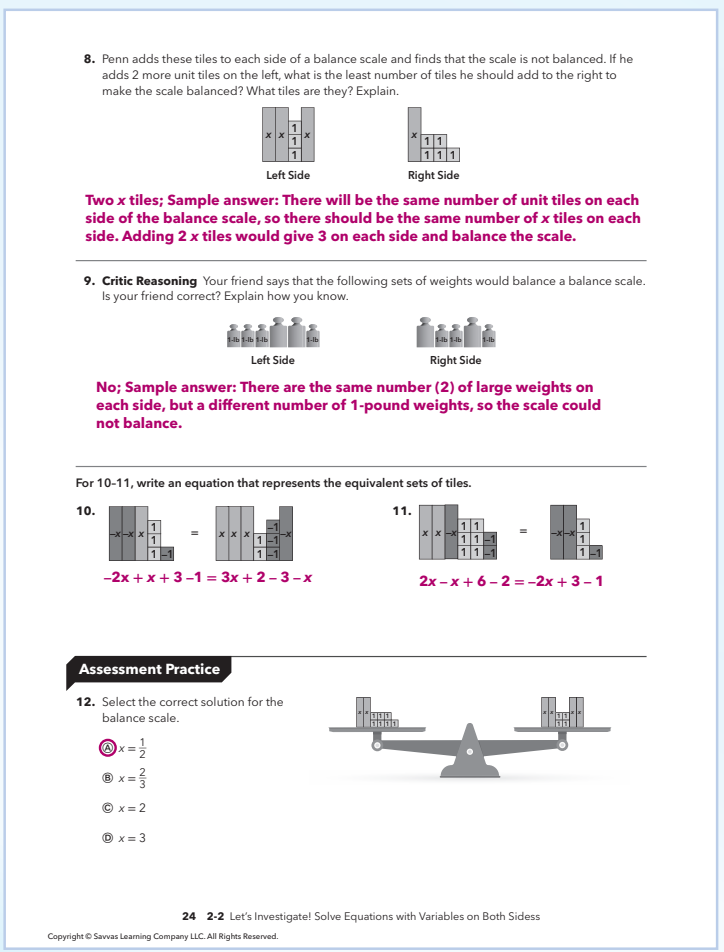

8. Penn adds these tiles to each side of a balance scale and finds that the scale is not balanced. If he adds 2 more unit tiles on the left, what is the least number of tiles he should add to the right to make the scale balanced? What tiles are they? Explain.

Left Side Right Side

Two x tiles; Sample answer: There will be the same number of unit tiles on each side of the balance scale, so there should be the same number of x tiles on each side. Adding 2 x tiles would give 3 on each side and balance the scale.

9. **Critic Reasoning** Your friend says that the following sets of weights would balance a balance scale. Is your friend correct? Explain how you know.

Left Side Right Side

No; Sample answer: There are the same number (2) of large weights on each side, but a different number of 1-pound weights, so the scale could not balance.

For 10–11, write an equation that represents the equivalent sets of tiles.

10. $-2x + x + 3 - 1 = 3x + 2 - 3 - x$
11. $2x - x + 6 - 2 = -2x + 3 - 1$

Assessment Practice

12. Select the correct solution for the balance scale.

Ⓐ $x = \frac{1}{2}$
Ⓑ $x = \frac{2}{3}$
Ⓒ $x = 2$
Ⓓ $x = 3$

24 2-2 Let's Investigate! Solve Equations with Variables on Both Sides

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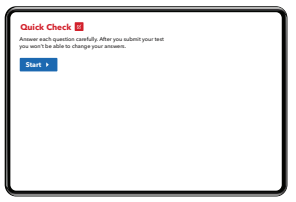
Item Analysis: Additional Practice

Items	DOK
1–3, 10, 11	1
4–7, 12	2
8, 9	3

MTP Quick Check

Formative Assessment

The Quick Check is available in the Assessment Sourcebook. The online Quick Check is automatically scored and appropriate remediation, practice, or enrichment will be assigned based on student performance.



Differentiation Library

The **Differentiation Library** at the beginning of this topic shows the differentiation options available, each of which can be appropriate for all students. However, based on the **Quick Check** results, you may prescribe:

0–3 points: Reteach to Build Understanding and Additional Vocabulary Support Activities
4–5 points: Enrichment Activity

In addition, **Multilingual Learners** may benefit from the Build Mathematical Literacy Activity.

Video Tutorials

Students can access instructional tutorials online.

- How Do You Solve an Equation with Variables on Both Sides?
- How Do You Solve a Word Problem Using an Equation with Variables on Both Sides?

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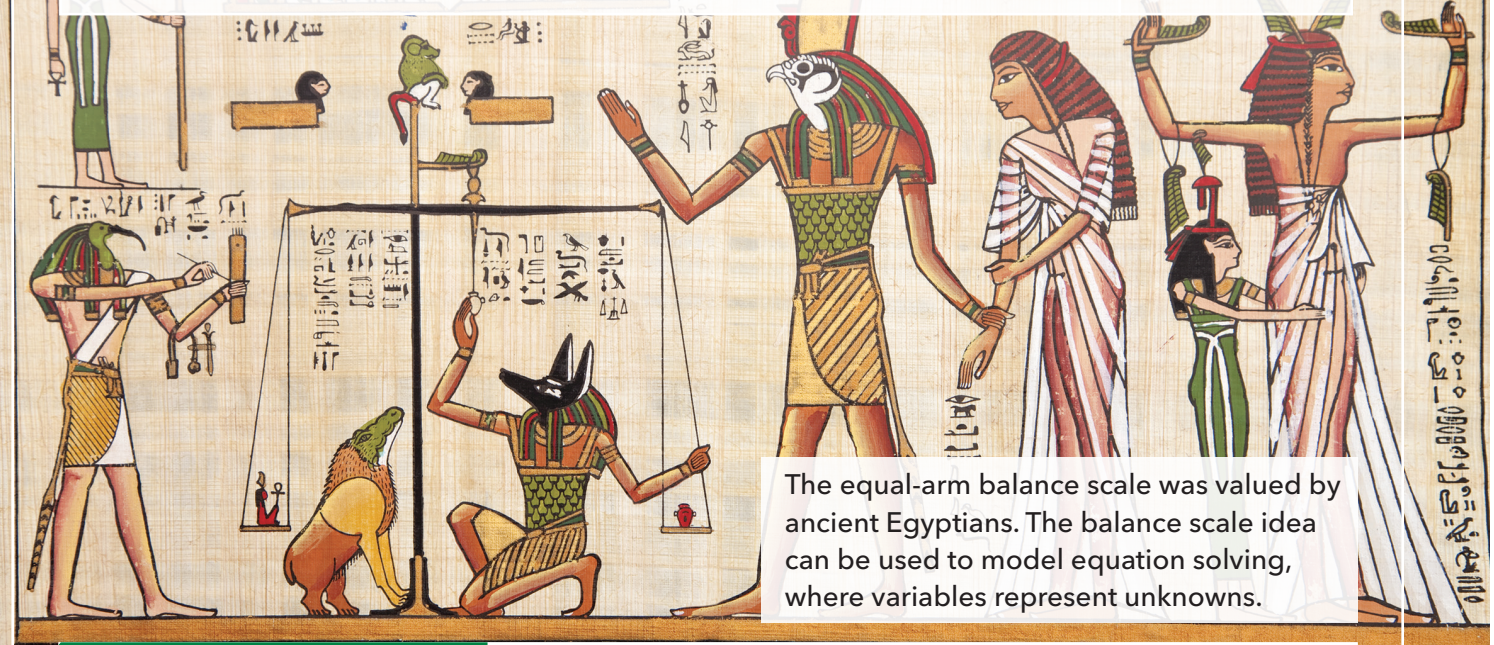
Solve Equations with Variables on Both Sides

Let's Investigate

Lesson 2-2

I can ... use inverse operations to solve equations with variables on both sides.



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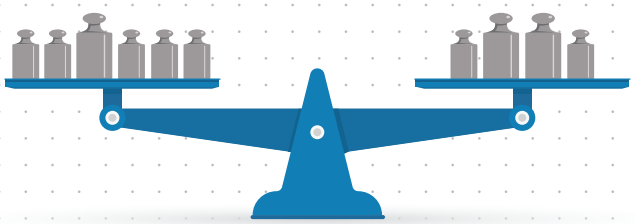
It All Balances Out

Reasoning The scale below shows two sizes of weights and is balanced. The small weights each have a mass of 1 lb. The mass of the large weights is unknown. What is the mass of one of the larger weights?



Write and use inverse operations and properties of equality to solve an algebraic equation that represents the problem. Explain.

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Create and draw your own scale problem for the class to solve. Make sure your problem has at least one variable on each side of the scale, and provide the solution.

Generalize How can you solve equations with variables on both sides of the equal sign? How can you check your answer to make sure it is correct?

Build G.R.I.T. Resilience
If your solution is incorrect, check each step in order for errors. Stay determined to find the correct solution.

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Choose a Problem

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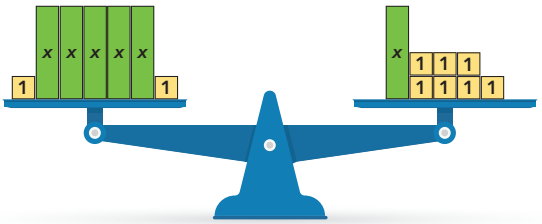
B Write and solve an algebraic equation to find the value of x.



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Connect

1. **Make Sense and Persevere** How was the strategy you used in *Choose a Problem* the same as your strategy in *It All Balances Out*? How were your strategies different?

2. **Use Structure** How can you turn a balance scale problem into an algebraic equation?

3. **Model with Math** What are some ways to represent an algebraic equation?

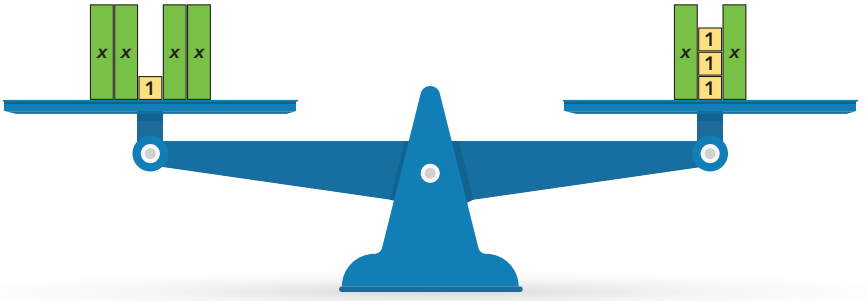
4. **Reasoning** What happens if the variable term is equivalent on both sides?

5. **Generalize** How is solving equations with variables on both sides different from solving with a variable on only one side?

Key Concept

You can use different representations, such as algebra tiles and balance scales, to model solving equations with variables on both sides.

$$4x + 1 = 2x + 3$$
$$4x - 2x + 1 = 2x - 2x + 3$$
$$2x + 1 = 3$$
$$2x + 1 - 1 = 3 - 1$$
$$2x = 2$$
$$2x \div 2 = 2 \div 2$$
$$x = 1$$



Talk About Ideas

How does adding or removing the same number of weights on each side of a balance scale connect to the properties of equality?

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Name _____

Lesson Help

Practice and Problem Solving

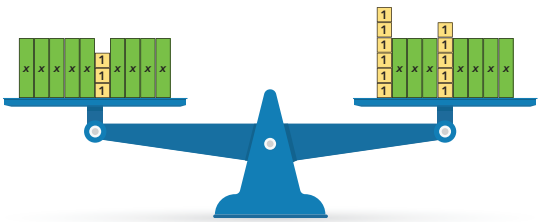
Use Structure For 6 and 7, find the mass of one large weight.

6.

7.

Use Structure For 8 and 9, find the value of one *x*-tile.

8.



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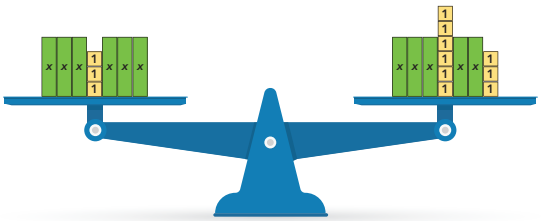
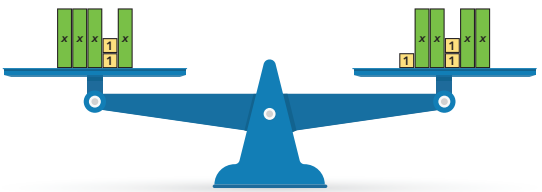
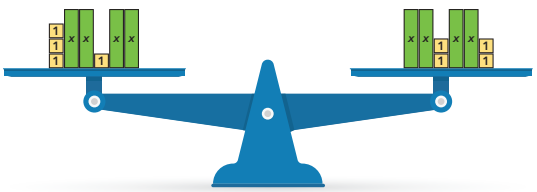


10. **Construct Arguments** Leylani uses a balance scale She places 3 *x* tiles, 3 unit tiles and 1 *x* tile on the left side and 1 unit tile, 2 *x* tiles, 2 unit tiles and 2 *x* tiles on the right side. Leylani says the scale is not balanced. Kelton says Leylani is incorrect and the scale is balanced. Who is correct? Explain.



11. **Critique Reasoning** If Khai removes 2 *x*-tiles from each side but also adds 1 unit tile to each side, will the scale still be balanced? Justify your reasoning.



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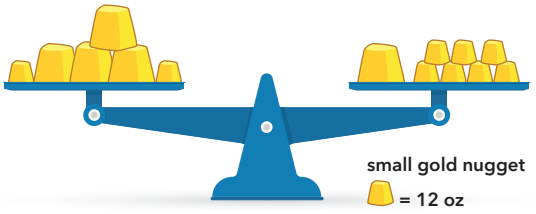
12. **Model with Math** When are pan balance scales an appropriate model to use?

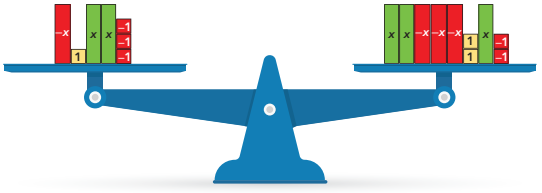
13. How can you tell when an equation has infinitely many solutions?

14. During the California Gold Rush 1848–1855, balance scales with copper pans were used to weigh gold. How many ounces of gold are in the large nuggets shown?



15. **Higher Order Thinking** Use the balance given scale.

a. Write an equation that represents the balance scale.

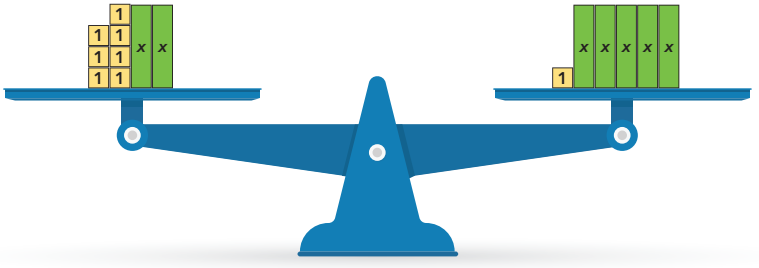


b. What do the negative algebra tiles represent?

16. **Use Appropriate Tools** For which algebraic equations are algebra tiles a useful tool?

Assessment Practice

17. Select the correct solution for the balance scale.



Ⓐ $x = \frac{1}{2}$

Ⓑ $x = \frac{4}{7}$

Ⓒ $x = 2$

Ⓓ $x = 3$

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