Grade 6

enVision® Mathematics

3-ACT MATH

Mathematical Modeling in 3 Acts

Take and Teach

SAVVAS

enVision™ 6-12

In Typical Textbook Problems...

Students are given a problem to solve.
The problem is already defined for students.

21. The principal at Concord High School's graduation reads names off a list at an average rate of eight names per minute. There are 288 students in the graduating class. How long will the ceremony last?

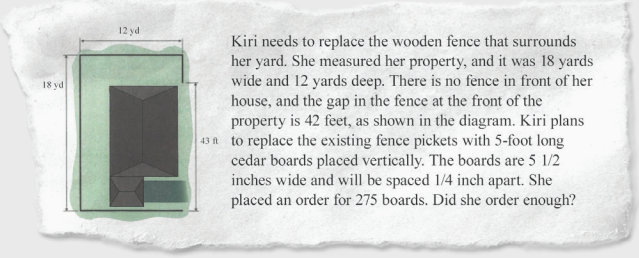
Students are given all the information required to solve the problem. Students are rarely required to determine all the information necessary to solve the problem because all the data needed to solve the problem is already provided for them.

21. The principal at Concord High School's graduation reads names off a list at an average rate of eight names per minute. There are 288 students in the graduating class. How long will the ceremony last?

Students flip to the back of the book to see if they got the right answer. If students don't get the correct answer, they ask for the formula to use (or where they went wrong with the formula) to solve the problem.

19. 15 Inches
21. 36 Minutes
23. 47.5 Feet

Students move onto another word problem that's just like the word problem they just solved.



Kiri needs to replace the wooden fence that surrounds her yard. She measured her property, and it was 18 yards wide and 12 yards deep. There is no fence in front of her house, and the gap in the fence at the front of the property is 42 feet, as shown in the diagram. Kiri plans to replace the existing fence pickets with 5-foot long cedar boards placed vertically. The boards are 5 1/2 inches wide and will be spaced 1/4 inch apart. She placed an order for 275 boards. Did she order enough?

Typical math textbook word problems from Dan Meyer's 2012 MISA talk

In Mathematical Modeling in 3 Acts...

Algebra 1 Act 1 Video

The Hook:

Students watch a video that prompts them to ask questions—in this case, "Will the shot go in?" Rather than being given all information up front, students actively define the word problem they are going to solve. **Act One** provides an entry point for every student, no matter their level of mathematical proficiency.

Algebra 1 Act 2 Video

Modeling with Math:

Students model with mathematics. Here, students figure out how they can determine if the shot will go in the basket. Students apply the mathematical concepts learned in the chapter and select the appropriate tools to solve the problem they defined in **Act One**. This is closer to the work of STEM professionals—it's both more challenging, and more fun.

Algebra 1 Act 3 Video

The Solution:

Students watch a video where the answer to the problem is revealed. In **Act Three**, students root for their conjectures and analyze their results. This act of analyzing and justifying solutions embodies the spirit of the Standards for Mathematical Practice.

SEQUEL

As students finish, tell them that the athlete makes another shot, this time from half court. Ask them to describe a shot that goes in and a shot that misses. [Answers will vary. Student's descriptions should include the line of symmetry and enough points to determine whether the shot goes in.]

Algebra 1 Sequel Problem

Sequel:

Students can extend their learning with a **Sequel** problem. The **Sequel** is a reimagined word problem related to the original problem, which requires students to model with mathematics.

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Mathematical Modeling in 3 Acts

Mathematical Modeling in 3 Acts is accessed through Realize, an easy-to-navigate learning platform. Access the teacher and student support pieces in print or online at SavvasRealize.com. Parallel print and digital support is available.

ACT TWO *Continued*

INTERESTING MOMENTS WITH STUDENTS

- **Before showing the answers, find out where there is disagreement.** Allow students to argue their cases and update their conjectures if they want to.
- **Students may conclude you only need to reflect one point to find out if the ball goes in.** They can reflect the basketball hoop across the line of symmetry and see if it reflects onto a basketball on the other side of the line. Invite any student who uses this method to explain it during the third act of the task. This may lead to a conversation about how Shot 5 is a counterexample to this conjecture.

ACT THREE **THE SOLUTION**

Play the video. The final part of the video shows the entirety of the six shots. The outcome of each shot is revealed. After each shot, ask students for a show of hands of whose initial conjecture was accurate and give them some praise.

MAIN QUESTION ANSWER

Shot 1: The ball goes in – a swish.
Shot 2: The ball goes in – off the backboard.
Shot 3: The ball misses – short.
Shot 4: The ball goes in.
Shot 5: The ball looks like it's going in, but then it hits the left side of the rim and bounces away.
Shot 6: The ball misses. It hits the back of the rim.

Do the "post-game" analysis. Help students understand why the result of the shot may not match what the parabola predicts. Students should focus on how the paths of the shots are not perfect parabolas. Students may suggest reasons the paths are not quite symmetric, such as gravity and air resistance. Celebrate any creative ideas that students offer to improve the method for predicting whether each shot will go in.

ONE POSSIBLE SOLUTION

For each shot, draw the line of symmetry. Then reflect the points shown across the line to see whether or not the shot goes in.

Using this method, Shots 1, 4, and 5 will go in, Shots 2 and 6 will be too long, and Shot 3 will be too short.

INTERESTING MOMENTS WITH STUDENTS

Shots 2 and 5 are designed to start a discussion. Students can discuss how the backboard influences the path of the ball for Shot 2. Meanwhile, Shot 5 may look like it will go in, but basketball shots do not necessarily travel in the plane shown in the video.

SEQUEL

As students finish, tell them that the athlete makes another shot, this time from half court. Ask them to describe a shot that goes in and a shot that misses. [Answers will vary. Students' descriptions should include the line of symmetry and enough points to determine whether the shot goes in.]

Sample of Teacher Support

Components

- **Engaging Multimedia**
  Mathematical Modeling in 3 Acts Video for every enVision 6-12 chapter.
- **Robust Teacher Support**
  Teacher support for every Mathematical Modeling in 3 Acts Task
- **Student Support**
  Accompanying student support for every Mathematical Modeling in 3 Acts Task is available to aid students with solution planning, analyzing conjectures, and justifying results.

Our mathematics modeling lessons engage students by featuring storytelling and linking classroom mathematics to everyday life, work, and decision-making.

– Eric Milou
enVision 6-12 Author

Bottom line: We are privileged ambassadors of a message that math models your world. We're in a privileged position, and we've got to make good on it.

– Dan Meyer, 2012
"Why Students Hate Word Problems" MISA talk

Try this sample lesson!

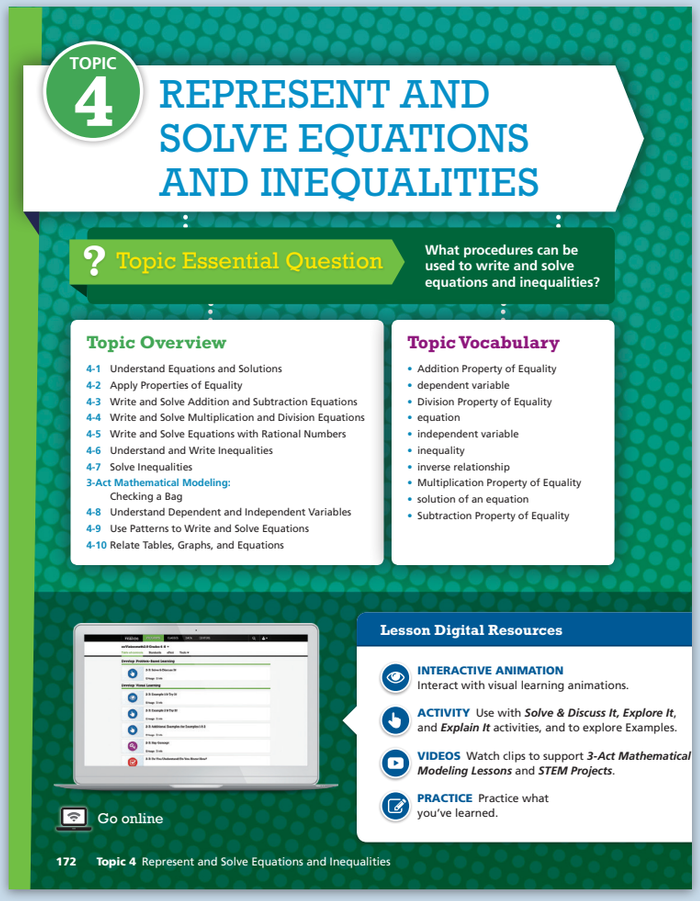
Visit Savvas.com/3am to download additional lesson resources and videos.

TOPIC 4

Topic Opener

Represent and Solve Equations and Inequalities

Available Online



Topic Essential Question

What procedures can be used to write and solve equations and inequalities?

Revisit the Topic Essential Question throughout the topic. See the Teacher's Edition for the Topic Review for notes about answering the question.

3-Act Mathematical Modeling

Generate excitement about the upcoming 3-Act Mathematical Modeling lesson by having students read about the math modeling problem for this topic.

See the Teacher's Edition lesson support for notes about how to use the lesson video in your classroom.

3-Act Mathematical Modeling: Checking a Bag

Video Activity

Lesson Overview

FOCUS

Objective

Students will be able to:

- ✔ use mathematical modeling to represent a problem situation and to propose a solution.
- ✔ test and verify the appropriateness of their math models.
- ✔ explain why the results from their mathematical models may not align exactly to the problem situation.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real-world situation exactly.

COHERENCE

Earlier in this topic, students:

- used properties of equality to write and solve equations.

In this lesson, students:

- develop a model to represent and propose a solution to a problem situation involving a one-step inequality.

Later in this course, students will:

- refine their mathematical modeling skills.

RIGOR

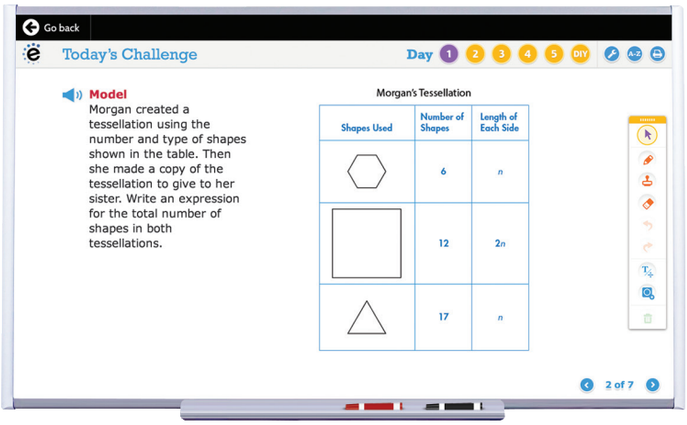
This mathematical modeling lesson focuses on application of both math content and math practices and processes.

- Students draw on their understanding of equality and inequality concepts to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

Math Anytime

Today's Challenge

Use the Topic 4 problems any time during this topic.



Mathematics Overview

In this lesson, students will develop and use a mathematical model to represent and propose a solution to a real-world problem involving a one-step inequality. Students will reinforce both their procedural skills as well as their understanding of the limitations of some mathematical models for real-world situations.

As students carry out mathematical modeling, they will also engage in sense-making, abstract and quantitative reasoning, and mathematical communication and argumentation. In testing and validating their models, students look for patterns and structure.

Applying Math Practices

Model with Math

The focus of this lesson is on mathematical modeling. To solve the problem situation presented, students will identify variables and the relationship among variables, develop a model that represents the situation, and use the model to propose a solution.

3-Act Mathematical Modeling

Video

ACT 1 The Hook

Students will be tasked with determining how many shoes someone can pack in a suitcase.

Play the Video and Brainstorm Questions

Have students complete **Question 1**. Encourage them to consider the situation and ask any questions that arise. Listen for interesting mathematical *and* non-mathematical questions. Ask students what makes each question interesting.

Q: What questions do you have? [Sample answer: Where is he going? Why does he needs so many shoes? How many shoes fit inside the suitcase? How much will that bag weigh?]

Pose the Main Question

After the question brainstorming, pose the Main Question students will be tasked with answering. Have students complete **Question 2**.

Main Question

Q: How many pairs of shoes can he pack?

Ask about Predictions

Have students complete **Questions 3–5**. You can survey the class for the range of predictions.

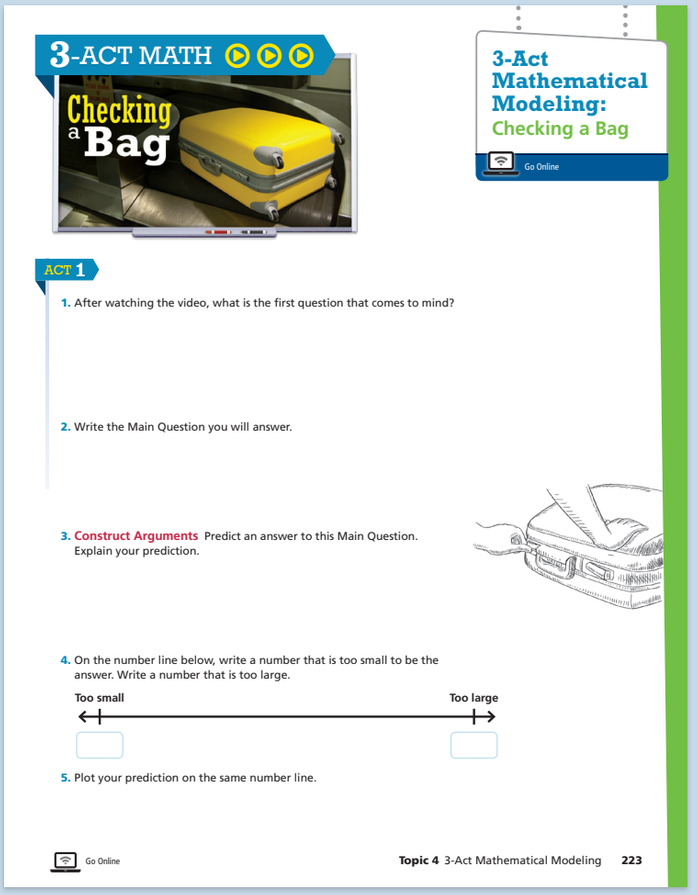
Q: Why do you think your prediction is the answer to the Main Question?

Q: Who had a similar prediction?

Q: How many of you agree with that prediction?

Q: Who has a different prediction?

Available Online


3-ACT MATH

Checking a Bag

3-Act Mathematical Modeling: Checking a Bag

Go Online

ACT 1

1. After watching the video, what is the first question that comes to mind?

2. Write the Main Question you will answer.

3. Construct Arguments Predict an answer to this Main Question. Explain your prediction.

4. On the number line below, write a number that is too small to be the answer. Write a number that is too large.

Too small — Too large

5. Plot your prediction on the same number line.

Go Online

Topic 4 3-Act Mathematical Modeling 223

ACT 2 The Model

Identify Important Info

Have students complete **Question 6**.

Q: What information would be helpful to solve the problem? [Sample answer: What the weight limit is; how much a pair of shoes weighs; how much the suitcase weighs]

Q: How could you get that information?

Q: Why do you need that information?

Reveal the Information

Reveal the information provided below using the online interactivity. Have students record information in **Question 7**.

Weight limit: 50 lb
Shoe weights: 2.2 lb, 2.5 lb, 2.2 lb, 2.3 lb
Suitcase weight: 6.2 lb

Develop a Model

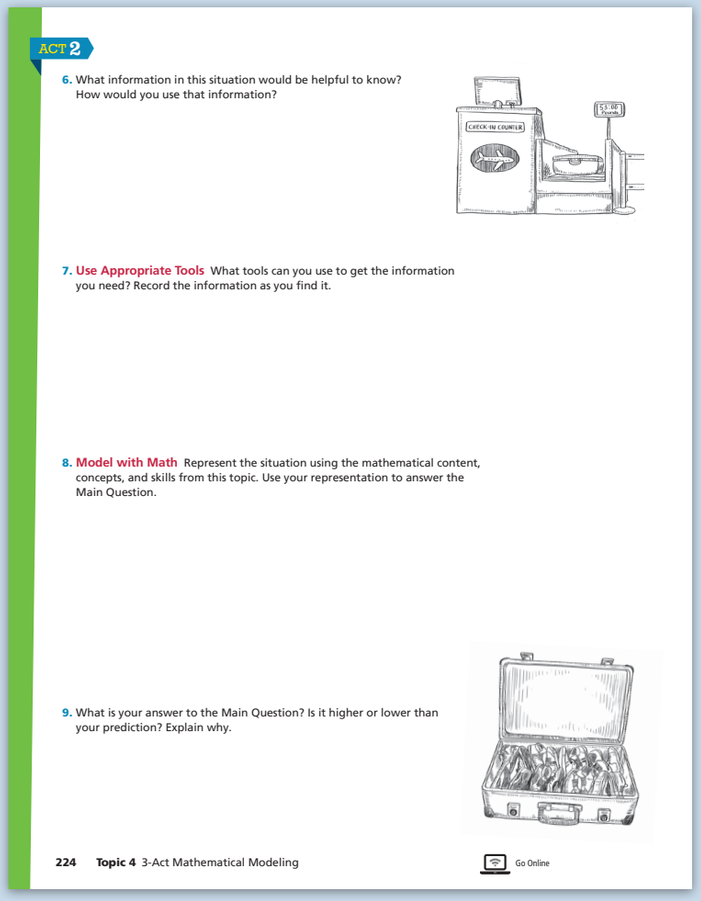
As students answer **Questions 8** and **9**, look at methods that they are using and prompt them to think about whether they need an equation or an inequality.

Q: How can you describe the relationship between the number of shoes and the weight limit? [Write an inequality; The weight of the shoes plus the weight of the suitcase must be no more than 50 pounds.]

Use the Model to Propose a Solution

After students answer **Questions 8** and **9**, facilitate a discussion about solution methods. If needed, project the possible student solutions (shown below).

Available Online



ACT 2

6. What information in this situation would be helpful to know? How would you use that information?

7. **Use Appropriate Tools** What tools can you use to get the information you need? Record the information as you find it.

8. **Model with Math** Represent the situation using the mathematical content, concepts, and skills from this topic. Use your representation to answer the Main Question.

9. What is your answer to the Main Question? Is it higher or lower than your prediction? Explain why.

224 **Topic 4** 3-Act Mathematical Modeling

Go Online

Possible Student Solutions

Karina's Work

p = number of pair of shoes

$2.3p + 6.2 \leq 50$

$2.3p \leq 43.8$

$p \leq 19.04$

He can pack 19 pairs.

Karina uses an average weight to write and solve a two-step inequality.

Malik's Work

Subtract the bag

$50 - 6.2 = 43.8$

$2.5x \leq 43.8$

$x \leq 17.52$

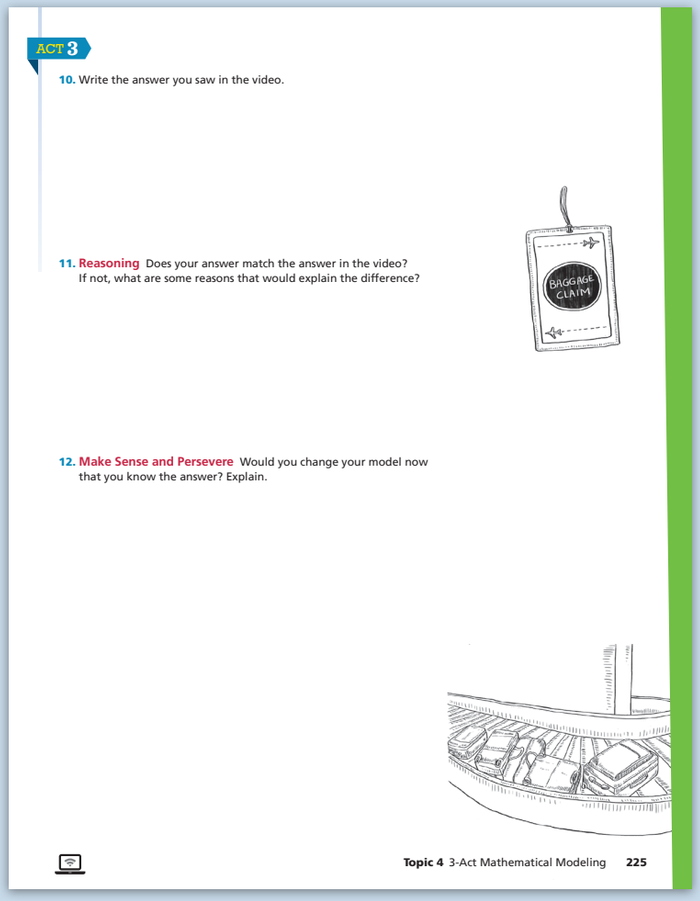
The answer is 17.52

Malik uses subtraction and the greatest weight to write a one-step inequality. He does not round his answer.

Video

ACT 3 The Solution and Sequel

Available Online

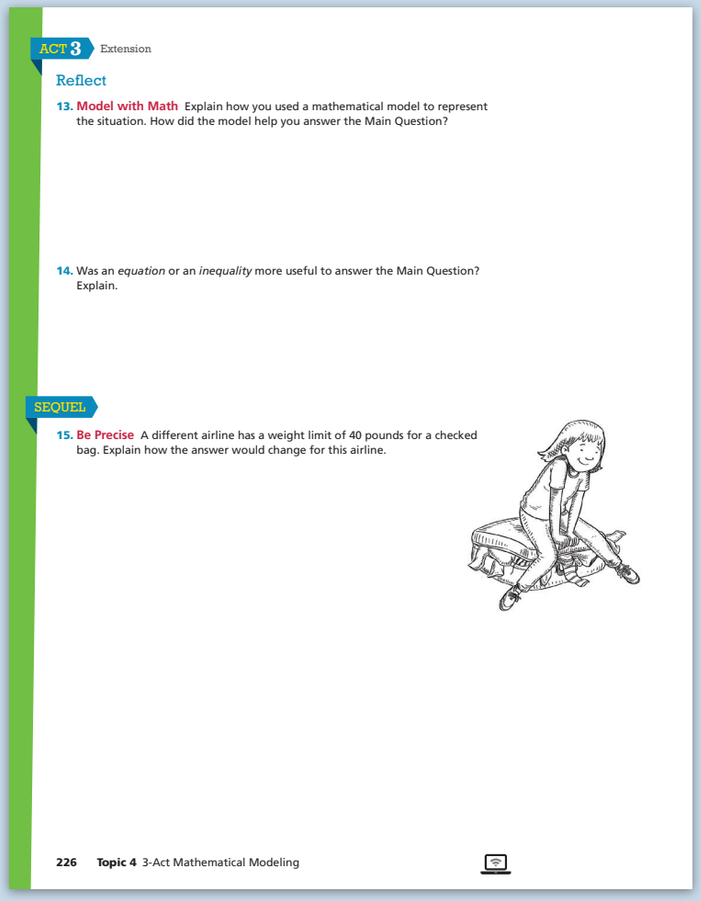


ACT 3

10. Write the answer you saw in the video.

11. **Reasoning** Does your answer match the answer in the video? If not, what are some reasons that would explain the difference?

12. **Make Sense and Persevere** Would you change your model now that you know the answer? Explain.



ACT 3 Extension

Reflect

13. **Model with Math** Explain how you used a mathematical model to represent the situation. How did the model help you answer the Main Question?

14. Was an *equation* or an *inequality* more useful to answer the Main Question? Explain.

SEQUEL

15. **Be Precise** A different airline has a weight limit of 40 pounds for a checked bag. Explain how the answer would change for this airline.

Use the Video to Reveal the Answer

The final part of the video shows the entire process of packing shoes and weighing the suitcase. Have students complete **Question 10**. Offer praise to the students who were closest to the actual answer.

Main Question Answer

19 pairs of shoes

Validate Conclusions

After students complete **Questions 11** and **12**, encourage them to discuss possible sources of error inherent in using math to model real-world situations. Look for students to point out that their models are still useful even though they are not perfect.

Q: Why does your answer not match the answer in the video? [Sample answer: Each pair of shoes has a slightly different weight. The ones we didn't weigh must have been lighter.]

Q: How useful was your model at predicting the answer?

Q: How could your model better represent the situation?

Reflect on Thinking

Use Appropriate Tools If time allows, have students complete **Questions 13** and **14** as an extension. Use this opportunity to discuss how students incorporate mathematical processes during the task.

Pose the Sequel

Be Precise Use **Question 15** to present a similar problem situation involving inequalities. You can assign to early finishers or as homework so students can test the usefulness of their models.

Q: A different airline has a weight limit of 40 pounds for a checked bag. Explain how the answer would change for this airline.

Using their models and the answer in the video, look for student solutions around 13 or 14 pairs of shoes.

Q: If the weight limit were 100 pounds, would your answer be twice the answer in the video? [No; Sample answer: Twice as many pairs of shoes wouldn't fit in the suitcase.]

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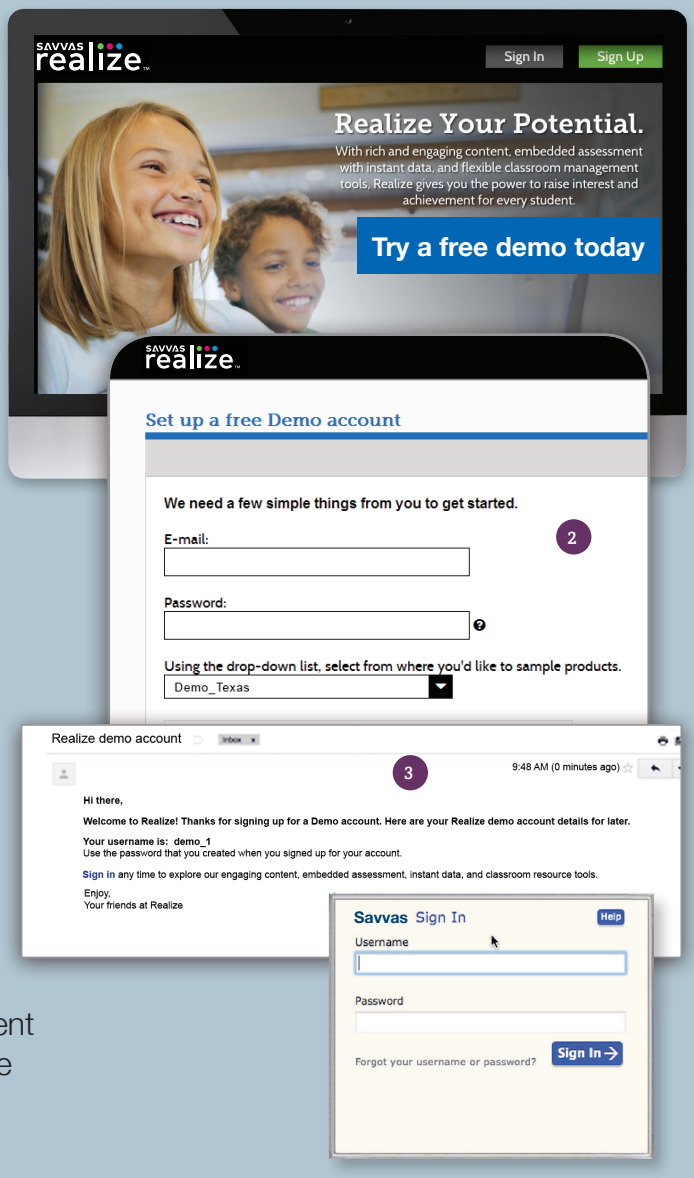

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