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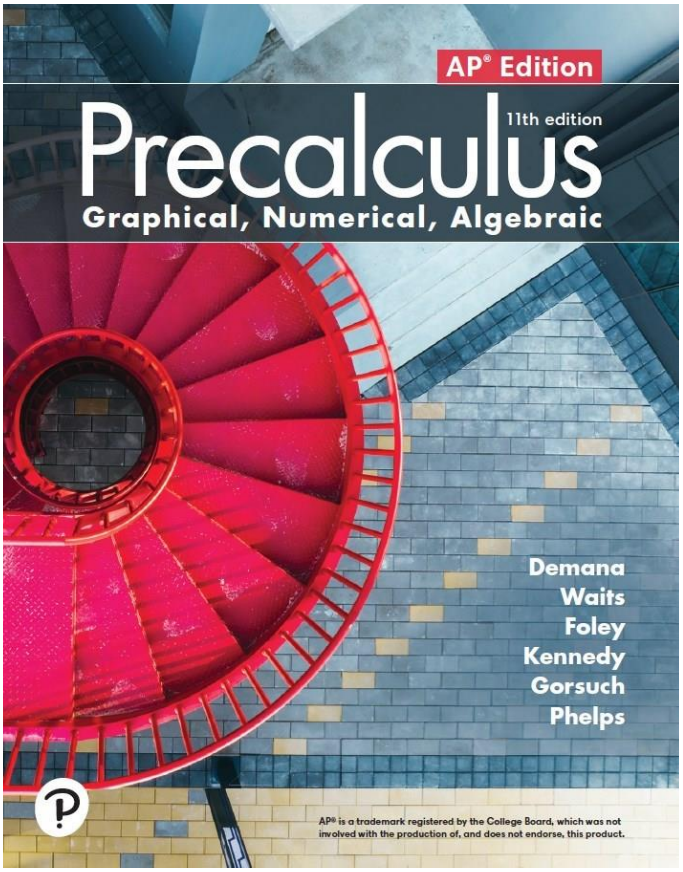

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to the

Advanced Placement Precalculus Course Frameworks Effective Fall 2023

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AP Precalculus Course Frameworks, by the College Board.

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1.7.A.3 If the polynomial in the numerator dominates the polynomial in the denominator for input values of large magnitude, then the quotient of the leading terms is a nonconstant polynomial, and the original rational function has the end behavior of that polynomial. If that polynomial is linear, then the graph of the rational function has a slant asymptote parallel to the graph of the line.	Lesson 2.6, pp. 211-223
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1.13.A.7 A piecewise-defined function consists of a set of functions defined over nonoverlapping domain intervals and is useful for modeling a data set or contextual scenario that demonstrates different characteristics over different intervals.	Lesson 1.4, pp. 94-104; Lesson 1.7, pp. 125-138
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UNIT 2 Exponential and Logarithmic Functions	
Topic 2.1 Change in Arithmetic and Geometric Sequences	
2.1.A Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers.	
2.1.A.1 A sequence is a function from the whole numbers to the real numbers. Consequently, the graph of a sequence consists of discrete points instead of a curve.	Lesson 3.1, pp. 242-256
2.1.A.2 Successive terms in an arithmetic sequence have a common difference, or constant rate of change.	Lesson 3.1, pp. 242-256
2.1.A.3 The general term of an arithmetic sequence with a common difference *d* is denoted by a_n and is given by $an = a_0 + d_n$, where a_0 is the initial value, or by $a_n = a_k + d(n - k)$, where a_k is the *kth* term of the sequence.	Lesson 3.1, pp. 242-256
2.1.B Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.	
2.1.B.1 Successive terms in a geometric sequence have a common ratio, or constant proportional change.	Lesson 3.1, pp. 242-256
2.1.B.2 The general term of a geometric sequence with a common ratio *r* is denoted by g_n and is given by $g_n = g_0r^n$, where g_0 is the initial value, or by $g_n = g_kr^{(n-k)}$, where *gk* is the *kth* term of the sequence.	Lesson 3.1, pp. 242-256
2.1.B.3 Increasing arithmetic sequences increase equally with each step, whereas increasing geometric sequences increase by a larger amount with each successive step.	Lesson 3.1, pp. 242-256
Topic 2.2 Change in Linear and Exponential Functions	
2.2.A Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.	
2.2.A.1 Linear functions of the form $f(x) = b + mx$ are similar to arithmetic sequences of the form, $a_n = a_0 + dn$, as both can be expressed as an initial value (*b or* a_0) plus repeated addition of a constant rate of change, the slope (*m or d*).	Lesson 3.1, pp. 242-256
2.2.A.2 Similar to arithmetic sequences of the form $a_n = a_k + d(n - k)$, which are based on a known difference, *d*, and a *kth* term, linear functions can be expressed in the form $f(x) = yi + m(x - xi)$ based on a known slope, *m*, and a point, (xi, yi).	Lesson 3.1, pp. 242-256
2.2.A.3 Exponential functions of the form $f(x) = ab^x$ are similar to geometric sequences of the $g_n = g_0r^n$, as both can be expressed as an initial value (a or g_0) times repeated multiplication by a constant proportion (b or r).	Lesson 3.1, pp. 242-256; Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
2.2.A.4 Similar to geometric sequences of the form $g_n = g_k - g_k\, r^{(n-k)}$), which are based on a known ratio, r, and a kth term, exponential functions can be expressed in the form $f(x) = y_i\, r^{(x-x_i)}$ based on a known ratio, r, and a point, (x_i, y_i).	Lesson 3.3, pp. 267-275
2.2.A.5 Sequences and their corresponding functions may have different domains.	Lesson 3.1, pp. 242-256; Lesson 3.3, pp. 267-275
2.2.B Describe similarities and differences between linear and exponential functions.	
2.2.B.1 Over equal-length input-value intervals, if the output values of a function change at constant rate, then the function is linear; if the output values of a function change proportionally, then the function is exponential	Lesson 3.1, pp. 242-256; Lesson 3.3, pp. 267-275

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2.2.B.2 Linear functions of the form $f(x) = b + mx$ and exponential functions of the form $f(x) = ab^x$ can both be expressed analytically in terms of an initial value and a constant involved with change. However, linear functions are based on addition, while exponential functions are based on multiplication.	Lesson 3.1, pp. 242-256; Lesson 3.3, pp. 267-275
2.2.B.3 Arithmetic sequences, linear functions, geometric sequences, and exponential functions all have the property that they can be determined by two distinct sequence or function values.	Lesson 3.1, pp. 242-256; Lesson 3.3, pp. 267-275
Topic 2.3 Exponential Functions	
2.3.A Identify key characteristics of exponential functions.	
2.3.A.1 The general form of an exponential function is $f(x) = ab^x$, with the *initial value a*, where a ≠ 0, and the base *b*, where $b > 0$, and b ≠ 1. When $a > 0$ and $b > 1$, the exponential function is said to demonstrate exponential growth. When $a > 0$ and $0 < b < 1$, the exponential function is said to demonstrate exponential decay.	Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
2.3.A.2 When the natural numbers are input values in an exponential function, the input value specifies the number of factors of the base to be applied to the function's initial value. The domain of an exponential function is all real numbers.	Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
2.3.A.3 Because the output values of exponential functions in general form are proportional over equal-length input-value intervals, exponential functions are always increasing or always decreasing, and their graphs are always concave up or always concave down. Consequently, exponential functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.	Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
2.3.A.4 If the values of the additive transformation function $g(x) = f(x) + k$ of any function f are proportional over equal-length input-value intervals, then f is exponential.	Lesson 3.3, pp. 267-275
2.3.A.5 For an exponential function in general form, as the input values increase or decrease without bound, the output values will increase or decrease without bound or will get arbitrarily close to zero. That is, for an exponential function in general form, $\lim_{x\to\pm\infty} ab^x = \infty$ $\lim_{x\to\pm\infty} ab^x = -\infty$ or $\lim_{x\to\pm\infty} ab^x = 0$	Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
Topic 2.4 Exponential Function Manipulation	
2.4.A Rewrite exponential expressions in equivalent forms.	
2.4.A.1 The product property for exponents states that $b^m b^n = b^{(m+n)}$. Graphically, this property implies that every horizontal translation of an exponential function, $f(x) = b^{(x+k)}$, is equivalent to a vertical dilation, $f(x) = b^{(x+k)} = b^x b^k = ab^x$, where $a = b^k$.	Lesson 3.2, pp. 257-265
2.4.A.2 The power property for exponents states that $(b^m)^n = b^{(mn)}$. Graphically, this property implies that every horizontal dilation of an exponential function, $f(x) = b^{(cx)}$, is equivalent to a change of the base of an exponential function, $f(x) = (b^c)^x$, where b^c is a constant and c ≠ 0.	Lesson 3.2, pp. 257-265

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2.4.A.3 The negative exponent property states that $b^{-n} = \frac{1}{b^n}$	Lesson 3.2, pp. 257-265
2.4.A.4 The value of an exponential expression involving an exponential unit fraction, such as $b^{(1/k)}$ where *k* is a natural number, is the *kth* root of *b*, when it exists.	Lesson 3.2, pp. 257-265
Topic 2.5 Exponential Function Context and Data Modeling	
2.5.A Construct a model for situations involving proportional output values over equal-length input-value intervals.	
2.5.A.1 Exponential functions model growth patterns where successive output values over equal length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.	Lesson 3.3, pp. 267-275
2.5.B.2 A constant may need to be added to the dependent variable values of a data set to reveal a proportional growth pattern.	Lesson 3.3, pp. 267-275
2.5.B.3 An exponential function model can be constructed from an appropriate ratio and initial value or from two input-output pairs. The initial value and the base can be found by solving a system of equations resulting from the two input-output pairs.	Lesson 3.3, pp. 267-275
2.5.A.4 Exponential function models can be constructed by applying transformations to $f(x) = ab^x$ based on characteristics of a contextual scenario or data set.	Lesson 3.3, pp. 267-275
2.5.A.5 Exponential function models can be constructed for a data set with technology using exponential regressions.	Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275
2.5.A.6 The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios.	Lesson 3.3, pp. 267-275
2.5.B Apply exponential models to answer questions about a data set or contextual scenario.	
2.5.B.1 For an exponential model in general form $f(x) = ab^x$, the base of the exponent, *b*, can be understood as a growth factor in successive unit changes in the input values and is related to a percent change in context.	Lesson 3.2, pp. 257-265. Lesson 3.3, pp. 267-275. Lesson 3.7, pp. 303-313
2.5.B.2 Equivalent forms of an exponential function can reveal different properties of the function. For example, if *d* represents number of days, then the base of $f(d) = 2^d$ indicates that the quantity increases by a factor of 2 every day, but the equivalent form $f(d) = (2^7)^{(d/7)}$ indicates that the quantity increases by a factor of 2^7 every week.	Lesson 3.3, pp. 267-275
2.5.B.3 Exponential models can be used to predict values for the dependent variable, depending on the contextual constraints on the domain.	Lesson 3.2, pp. 257-265. Lesson 3.3, pp. 267-275. Lesson 3.7, pp. 303-313
Topic 2.6 Competing Function Model Validation	
2.6.A Construct linear, quadratic, and exponential models based on a data set.	
2.6.A.1 Two variables in a data set that demonstrate a slightly changing rate of change can be modeled by linear, quadratic, and exponential function models.	Lesson 1.7, pp. 125-138; Lesson 3.2, pp. 257-265; Lesson 3.3, pp. 267-275; Lesson 3.7, pp. 303-313

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2.6.A.2 Models can be compared based on contextual clues and applicability to determine which model is most appropriate.	Lesson 1.7, pp. 125-138; Lesson 3.3, pp. 267-275
2.6.B Validate a model constructed from a data set.	
2.6.B.1 A model is justified as appropriate for a data set if the graph of the residuals of a regression, the residual plot, appear without pattern.	Lesson 2.2, pp. 161-174; Lesson 3.3, pp. 267-275
2.6.B.2 The difference between the predicted and actual values is the error in the model. Depending on the data set and context, it may be more appropriate to have an underestimate or overestimate for any given interval.	Lesson 2.2, pp. 161-174; Lesson 3.3, pp. 267-275; Lesson 3.7, pp. 303-313
Topic 2.7 Composition of Functions	
2.7.A Evaluate the composition of two or more functions for given values.	
2.7.A.1 If f and g are functions, the composite function $f \circ g$ maps a set of input values to a set of output values such that the output values of g are used as input values of f. For this reason, the domain of the composite function is restricted to those input values of g for which the corresponding output value is in the domain of f. $(f \circ g)(x)$ can also be represented as $f(g(x))$.	Lesson 1.4, pp. 94-104
2.7.A.2 Values for the composite function $f \circ g$ can be calculated or estimated from the graphical, numerical, analytical, or verbal representations of f and g by using output values from g as input values for f.	Lesson 1.4, pp. 94-104
2.7.A.3 The composition of functions is not commutative; that is, $f \circ g$ and $g \circ f$ are typically different functions; therefore, $f(g(x))$ and $g(f(x))$ are typically different values.	Lesson 1.4, pp. 94-104
2.7.A.4 If the function $f(x) = x$ is composed with any function g, the resulting composite function is the same as g; that is, $g(f(x)) = f(g(x)) = g(x)$. The function $f(x) = x$ is called the identity function. When composing two functions, the identify function acts in the same way as 0, the additive identity, when adding two numbers and 1, the multiplicative identity, when multiplying two numbers.	Lesson 1.3, pp. 86-93; Lesson 1.4, pp. 94-104
2.7.B Construct a representation of the composition of two or more functions.	
2.7.B.1 Function composition is useful for relating two quantities that are not directly related by an existing formula.	Lesson 1.4, pp. 94-104
2.7.B.2 When analytic representations of the functions f and g are available, an analytic representation of $f(g(x))$ can be constructed by substituting $g(x)$ for every instance of x in f.	Lesson 1.4, pp. 94-104
2.7.B.3 A numerical or graphical representation of $f \circ g$ can often be constructed by calculating or estimating values for $(x, f(g(x)))$.	Lesson 1.4, pp. 94-104
2.7.C Rewrite a given function as a composition of two or more functions.	
2.7.C.1 Functions given analytically can often be decomposed into less complicated functions. When properly decomposed, the variable in one function should replace each instance of the function with which it was composed.	Lesson 1.4, pp. 94-104; Lesson 1.6, pp. 115-120

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2.7.C.2 An additive transformation of a function, f, that results in vertical and horizontal translations can be understood as the composition of $g(x) = x + k$ with f.	Lesson 1.6, pp. 115-120
2.7.C.3 A multiplicative transformation of a function, f, that results in vertical and horizontal dilations can be understood as the composition of $g(x) = kx$ with f.	Lesson 1.6, pp. 115-120
Topic 2.8 Inverse Functions	
2.8.A Determine the input-output pairs of the inverse of a function.	
2.8.A.1 On a specified domain, a function, f, has an inverse function, or is invertible, if each output value of f is mapped from a unique input value. The domain of a function may be restricted in many ways to make the function invertible.	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.A.2 An inverse function can be thought of as a reverse mapping of the function. An inverse function, f^{-1}, maps the output values of a function, f, on its invertible domain to their corresponding input values; that is, if $f(a) = b$, then $f^{-1}(b) = a$. Alternately, on its invertible domain, if a function consists of input-output pairs (a, b), then the inverse function consists of input-output pairs (b, a).	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.B Determine the inverse of a function on an invertible domain.	
2.8.B.1 The composition of a function, f, and its inverse function, f^{-1}, is the identity function; that is, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.B.2 On a function's invertible domain, the function's range and domain are the inverse function's domain and range, respectively. The inverse of the table of values of $y = f(x)$ can be found by reversing the input-output pairs; that is, (a, b) corresponds to (b, a).	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.B.3 The inverse of the graph of the function $y = f(x)$ can be found by reversing the roles of the $x-$ and $y-axes$; that is, by reflecting the graph of the function over the graph of the identity function $h(x) = x$.	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.B.4 The inverse of the function can be found by determining the inverse operations to reverse the mapping. One method for finding the inverse of the function f is reversing the roles of x and y in the equation $y = f(x)$, then solving for $f - y = 1(x)$.	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
2.8.B.5 In addition to limiting the domain of a function to obtain an inverse function, contextual restrictions may also limit the applicability of an inverse function.	Lesson 1.5, pp. 105-113; Lesson 3.4, pp. 276-286; Lesson 4.7, pp. 378-386
Topic 2.9 Logarithmic Expressions	
2.9.A Evaluate logarithmic expressions.	
2.9.A.1 The logarithmic expression $\log_b c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c. That is, $\log_b c = a$ if and only if $b^a = c$, where a and c are constants, $b > 0$, and $b \neq 1$. (when the base of a logarithmic expression is not specified, it is understood as the common logarithm with a base of 10)	Lesson 3.4, pp. 276-286
2.9.A.2 The values of some logarithmic expressions are readily accessible through basic arithmetic while other values can be estimated through the use of technology.	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292

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2.9.A.3 On a logarithmic scale, each unit represents a multiplicative change of the base of the logarithm. For example, on a standard scale, the units might be *0, 1, 2*, …, while on a logarithmic scale, using logarithm base *10*, the units might be $10^0, 10^1, 10^2$, ….	Lesson 3.6, pp. 293-302
Topic 2.10 Inverses of Exponential Functions	
2.10.A Construct representations of the inverse of an exponential function with an initial value of 1.	
2.10.A.1 The general form of a logarithmic function is $f(x) = a \log_b x$, with *base* b, where $b > 0$, $b \neq 1$, and $a \neq 0$.	Lesson 3.4, pp. 276-286
2.10.A.2 The way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Output values of general-form exponential functions change proportionately as input values increase in equal-length intervals. However, input values of general-form logarithmic functions change proportionately as output values increase in equal-length intervals. Alternately, exponential growth is characterized by output values changing multiplicatively as input values change additively, whereas logarithmic growth is characterized by output values changing additively as input values change multiplicatively.	Lesson 3.5, pp. 287-292; Lesson 3.6, pp. 293-302
2.10.A.3 $f(x) = \log_b x$ and $(g)x = b^2$, where $b > 0$ and $b \neq 1$, are inverse functions. That is, $g(f(x)) = f(g(x)) = x$.	Lesson 3.4, pp. 276-286
2.10.A.4 The graph of the logarithmic function $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$, is a reflection of the graph of the exponential function $g(x) = b^x$, where $b > 0$ and $b \neq 1$, over the graph of the identity function $h(x) = x$.	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292
2.10.A.5 If (*s*, *t*) is an ordered pair of the exponential function $g(x) = b^x$, where $b > 0$ and $b \neq 1$, then (*t*, *s*) is an ordered pair of the logarithmic function $f(x) = \log_b x$, where $b > 0$ and $b \neq 1$.	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292
Topic 2.11 Logarithmic Functions	
2.11.A Identify key characteristics of logarithmic functions.	
2.11.A.1The domain of a logarithmic function in general form is any real number greater than zero, and its range is all real numbers.	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292
2.11.A.2 Because logarithmic functions are inverses of exponential functions, logarithmic functions are also always increasing or always decreasing, and their graphs are either always concave up or always concave down. Consequently, logarithmic functions do not have extrema except on a closed interval, and their graphs do not have points of inflection.	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292
2.11.A.3 The additive transformation function *g(x) = f(x + k)*, where *k ≠ 0*, of a logarithmic function *f* in general form does not have input values that are proportional over equal length output-value intervals. However, if the input values of the additive transformation function, *g(x) = f(x + k)*, of any function f are proportional over equal-length output value intervals, then *f* is logarithmic.	Lesson 3.6, pp. 293-302; Lesson 3.7, pp. 303-313

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2.11.A.4 With their limited domain, logarithmic functions in general form are vertically asymptotic to *x = 0*, with an end behavior that is unbounded. That is, for a logarithmic function in general form, $\lim_{x\to 0^+} alog_b x = \pm\infty$ and $\lim_{x\to\infty} alog_b x = \pm\infty$	Lesson 3.4, pp. 276-286; Lesson 3.5, pp. 287-292
Topic 2.12 Logarithmic Function Manipulation	
2.12.A Rewrite logarithmic expressions in equivalent forms.	
2.12.A.1 The product property for logarithms states that $log_b(xy) = log_b x + log_b y$. Graphically, this property implies that every horizontal dilation of a logarithmic function, $f(x) = log_b(kx)$, is equivalent to a vertical translation, $f(x) = log_b(kx) = log_b k + log_b x = a + log_b x$, where $a = log_b k$.	Lesson 3.5, pp. 287-292
2.12.A.2 The power property for logarithms states that $log_b x^n = log_b x$. Graphically, this property implies that raising the input of a logarithmic function to a power, $f(x) = log_b x^k$, results in a vertical dilation, $f(x) = log_b x^k = klog_b x$.	Lesson 3.5, pp. 287-292
2.12.A.3 The change of base property for logarithms states that $log_b x = \frac{log_a x}{log_a b}$ where $a > 1$. and $a \neq 0$. This implies that all logarithmic functions are vertical dilations of each other.	Lesson 3.5, pp. 287-292
2.12.A.4 The function $f(x) = lnx$ is a logarithmic function with the natural base e; that is, $lnx = logex$.	Lesson 3.4, pp. 276-286
Topic 2.13 Exponential and Logarithmic Equations and Inequalities	
2.13.A Solve exponential and logarithmic equations and inequalities.	
2.13.A.1 Properties of exponents, properties of logarithms, and the inverse relationship between exponential and logarithmic functions can be used to solve equations and inequalities involving exponents and logarithms.	Lesson 3.7, pp. 303-313
2.13.A.2 When solving exponential and logarithmic equations found through analytical or graphical methods, the results should be examined for extraneous solutions precluded by the mathematical or contextual limitations.	Lesson 3.7, pp. 303-313
2.13.A.3 Logarithms can be used to rewrite expressions involving exponential functions in different ways that may reveal helpful information. Specifically, $b^x = c^{(logc\ b)(x)}$.	Lesson 3.7, pp. 303-313
2.13.B Construct the inverse function for exponential and logarithmic functions.	
2.13.B.1 The function $f(x) = ab^{(x-h)} + k$ is a combination of additive transformations of an exponential function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.	Lesson 3.4, pp. 276-286; Lesson 3.7, pp. 303-313
2.13.B.2 The function $f(x) = alog_b(x - h) + k$ is a combination of additive transformations of a logarithmic function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.	Lesson 3.4, pp. 276-286; Lesson 3.7, pp. 303-313

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Topic 2.14 Logarithmic Function Context and Data Modeling	
2.14.A Construct a logarithmic function model.	
2.14.A.1 Logarithmic functions are inverses of exponential functions and can be used to model situations involving proportional growth, or repeated multiplication, where the input values change proportionally over equal-length output-value intervals. Alternately, if the output value is a whole number, it indicates how many times the initial value has been multiplied by the proportion.	Lesson 3.4, pp. 276-286; Lesson 3.7, pp. 303-313
2.14.A.2 A logarithmic function model can be constructed from an appropriate proportion and a real zero or from two input-output pairs.	Lesson 3.7, pp. 303-313
2.14.A.3 Logarithmic function models can be constructed by applying transformations to $f\,(x) = alog_b\, x$ based on characteristics of a context or data set.	Lesson 3.7, pp. 303-313
2.14.A.4 Logarithmic function models can be constructed for a data set with technology using logarithmic regressions.	Lesson 3.7, pp. 303-313
2.14.A.5 The natural logarithm function is often useful in modeling real-world phenomena.	Lesson 3.7, pp. 303-313
2.14.A.6 Logarithmic function models can be used to predict values for the dependent variable.	Lesson 3.7, pp. 303-313
Topic 2.15 Semi-log Plots	
2.15.A Determine if an exponential model is appropriate by examining a semi-log plot of a data set.	
2.15.A.1 In a semi-log plot, one of the axes is logarithmically scaled. When the y-axis of a semi-log plot is logarithmically scaled, data or functions that demonstrate exponential characteristics will appear linear.	Lesson 3.6, pp. 293-302
2.15.A.2 An advantage of semi-log plots is that a constant never needs to be added to the dependent variable values to reveal that an exponential model is appropriate	Lesson 3.6, pp. 293-302
2.15.B Construct the linearization of exponential data.	
2.15.B.1 Techniques used to model linear functions can be applied to a semi-log graph.	Lesson 3.6, pp. 293-302
2.15.B.2 For an exponential model of the form $y = ab^x$, the corresponding linear model for the semi log plot is $y = (log_n\, b)x + log_n\, a$, where $n > 0$ and $n \neq 1$. Specifically, the linear rate of change is $log_n\, b$, and the initial linear value is $log_n\, a$.	Lesson 3.6, pp. 293-302
UNIT 3 Trigonometric and Polar Functions	
Topic 3.1 Periodic Phenomena	
3.1.A Construct graphs of periodic relationships based on verbal representations.	
3.1.A.1 A periodic relationship can be identified between two aspects of a context if, as the input values increase, the output values demonstrate a repeating pattern over successive equal-length intervals.	Lesson 4.3, pp. 336-348; Lesson 4.6, pp. 371-377
3.1.A.2 The graph of a periodic relationship can be constructed from the graph of a single cycle of the relationship.	Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361; Lesson 4.6, pp. 371-377

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3.1.B Describe key characteristics of a periodic function based on a verbal representation.	
3.1.B.1 The period of the function is the smallest positive value k such that $f\ (x + k) = f(x)$ for all x in the domain. Consequently, the behavior of a periodic function is completely determined by any interval of width k.	Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361
3.1.B.2 The period can be estimated by investigating successive equal-length output values and finding where the pattern begins to repeat.	Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361
3.1.B.3 Periodic functions take on characteristics of other functions, such as intervals of increase and decrease, different concavities, and various rates of change. However, with periodic functions, all characteristics found in one period of the function will be in every period of the function.	Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361; Lesson 4.6, pp. 371-377
Topic 3.2 Sine, Cosine, and Tangent	
3.2.A Determine the sine, cosine, and tangent of an angle using the unit circle.	
3.2.A.1 In the coordinate plane, an angle is in standard position when the vertex coincides with the origin and one ray coincides with the positive $x - axis$. The other ray is called the $terminal\ ray$. Positive and negative angle measures indicate rotations from the positive $x - axis$ in the counterclockwise and clockwise direction, respectively. Angles in standard position that share a terminal ray differ by an integer number of revolutions.	Lesson 4.1, pp. 320-326
3.2.A.2 The radian measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin subtended by the angle to the radius of that same circle. For a unit circle, which has radius 1, the radian measure is the same as the length of the subtended arc.	Lesson 4.1, pp. 320-326
3.2.A.3 Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The $sine$ of the angle is the ratio of the vertical displacement of P from the $x - axis$ to the distance between the origin and point P. Therefore, for a unit circle, the sine of the angle is the y-coordinate of point P.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
3.2.A.4 Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine of the angle is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and point P. Therefore, for a unit circle, the cosine of the angle is the x-coordinate of point P.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
3.2.A.5 Given an angle in standard position, the $tangent$ of the angle is the slope, if it exists, of the terminal ray. Because the slope of the terminal ray is the ratio of the vertical displacement to the horizontal displacement over any interval, the tangent of the angle is the ratio of the y-coordinate to the x-coordinate of the point at which the terminal ray intersects the unit circle; alternately, it is the ratio of the angle's sine to its cosine.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370

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pic 3.3 Sine and Cosine Function Values	
3.3.A Determine coordinates of points on a circle centered at the origin.	
3.3.A.1 Given an angle of measure θ in standard position and a circle with radius r centered at the origin, there is a point, P, where the terminal ray intersects the circle. The coordinates of point P are $rcos\theta, rsin\theta$	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
3.3.A.2 The geometry of isosceles right and equilateral triangles, while attending to the signs of the values based on the quadrant of the angle, can be used to find exact values for the cosine and sine of angles that are multiples $\frac{\pi}{4}$ and $\frac{\pi}{6}$ of radians and whose terminal rays do not lie on an axis.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
Topic 3.4 Sine and Cosine Function Graphs	
3.4.A Construct representations of the sine and cosine functions using the unit circle.	
3.4.A.1 Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine function, $f(\theta)$ = sinθ, gives the y-coordinate, or vertical displacement from the x-axis, of point P. The domain of the sine function is all real numbers.	Lesson 1.3, pp. 86-93; Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361
3.4.A.2 As the input values, or angle measures, of the sine function increase, the output values oscillate between -1 and 1, taking every value in between and tracking the vertical distance of points on the unit circle from the x-axis.	Lesson 1.3, pp. 86-93; Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
3.4.A.3 Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine function, $f(\theta) = cos\theta$, gives the x-coordinate, or horizontal displacement from the y-axis, of point P. The domain of the cosine function is all real numbers.	Lesson 1.3, pp. 86-93; Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348
3.4.A.4 As the input values, or angle measures, of the cosine function increase, the output values oscillate between -1 and 1, taking every value in between and tracking the horizontal distance of points on the unit circle from the y-axis.	Lesson 1.3, pp. 86-93; Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.4, pp. 349-361
Topic 3.5 Sinusoidal Functions	
3.5.A Identify key characteristics of the sine and cosine functions.	
3.5.A.1 A sinusoidal function is any function that involves additive and multiplicative transformations of $f(\theta+) = sin\ \theta$. The sine and cosine functions are both sinusoidal functions, with $cos\theta = \sin(\theta + \frac{\pi}{2})$	Lesson 4.4, pp. 349-361
3.5.A.2 The period and frequency of a sinusoidal function are reciprocals. The period of $f\ (\theta) = sin\theta$ and $g\ (\theta\ ) = cos\theta$ is 2π, and the frequency is $\frac{1}{2\pi}$	Lesson 4.4, pp. 349-361
3.5.A.3 The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The amplitude of $f\ (\theta\ ) = sin\theta$ and $g\ (\theta\ ) = cos\theta$ is 1.	Lesson 4.4, pp. 349-361
3.5.A.4 The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of $y = sin\theta$ and $y = cos\theta\ is\ y = 0$.	Lesson 4.4, pp. 349-361

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3.5.A.5 As input values increase, the graphs of sinusoidal functions oscillate between concave down and concave up.	Lesson 1.3, pp. 86-93; Lesson 4.4, pp. 349-361
3.5.A.6 The graph of $y = sin\theta$ has rotational symmetry about the origin and is therefore an odd function. The graph of $y = cos\theta$ has reflective symmetry over the y-axis and is therefore an even function.	Lesson 1.3, pp. 86-93; Lesson 4.4, pp. 349-361; Lesson 4.4, pp. 349-361; Lesson 5.1, pp. 395-402
Topic 3.6 Sinusoidal Function Transformations	
3.6.A Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	
3.6.A.1 Functions that can be written in the form $f(\theta) = a\sin(b(\theta + c)) + d$ or $g(\theta) = acos(b(\theta + c)) + d$, where $a, b, c,$ and d are real numbers and $a \neq 0$, are sinusoidal functions and are transformations of the sine and cosine functions. Additive and multiplicative transformations are the same for both sine and cosine because the cosine function is a phase shift of the sine function by $-\frac{\pi}{2}$ units.	Lesson 4.4, pp. 349-361; Lesson 4.6, pp. 371-377
3.6.A.2 The graph of the additive transformation $g(\theta) = sin\theta + d$ of the sine function $f(\theta) = sin\theta$ is a vertical translation of the graph of f, including its midline, by d units. The same transformation of the cosine function yields the same result.	Lesson 4.4, pp. 349-361
3.6.A.3 The graph of the additive transformation $g(\theta) = sin(\theta + c)$ of the sine function $f(\theta) = sin\theta$ is a horizontal translation, or phase shift, of the graph of f by $-c\ units$. The same transformation of the cosine function yields the same result.	Lesson 4.4, pp. 349-361
3.6.A.4 The graph of the multiplicative transformation $g(\theta) = a\sin\theta$ of the sine function $f(\theta) = sin\theta$ is a vertical dilation of the graph of f and differs in amplitude by a factor of |a|. The same transformation of the cosine function yields the same result.	Lesson 4.4, pp. 349-361
3.6.A.5 The graph of the multiplicative transformation $g(\theta) = sin(b\theta)$ of the sine function $f(\theta) = sin\theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\|\frac{1}{b}\|$. The same transformation of the cosine function yields the same result.	Lesson 4.4, pp. 349-361
3.6.A.6 The graph of $y = f(\theta) = asin(b(\theta + c)) + d$ has an amplitude of $\|a\|$ units, a period of $\|\frac{1}{b}\|2\pi$ units, a midline vertical shift of d units from $y = 0$, and a phase shift of $-c$ units. The same transformations of the cosine function yield the same results.	Lesson 4.4, pp. 349-361
Topic 3.7 Sinusoidal Function Context and Data Modeling	
3.7.A Construct sinusoidal function models of periodic phenomena.	
3.7.A.1 The smallest interval of input values over which the maximum or minimum output values start to repeat, that is, the input-value interval between consecutive maxima or consecutive minima, can be used to determine or estimate the period and frequency for a sinusoidal function model.	Lesson 4.4, pp. 349-361

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3.7.A.2 The maximum and minimum output values can be used to determine or estimate the amplitude and vertical shift for a sinusoidal function model.	Lesson 4.4, pp. 349-361
3.7.A.3 An actual pair of input-output values can be compared to pairs of input-output values produced by a sinusoidal function model to determine or estimate a phase shift for the model.	Lesson 4.4, pp. 349-361
3.7.A.4. Sinusoidal function models can be constructed for a data set with technology by estimating key values or using sinusoidal regressions.	Lesson 4.4, pp. 349-361
3.7.A.5 Sinusoidal functions that model a data set are frequently only useful over their contextual domain and can be used to predict values of the dependent variable from values of the independent variable.	Lesson 4.4, pp. 349-361
Topic 3.8 The Tangent Function	
3.8.A Construct representations of the tangent function using the unit circle.	
3.8.A.1 Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The tangent function, $f\ (\theta)\ =\ tan\theta$, gives the slope of the terminal ray.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370
3.8.A.2 Because the slope of the terminal ray is the ratio of the change in the y-values to the change in the x-values between any two points on the ray, the tangent function is also the ratio of the sine function to the cosine function. Therefore, $tan\theta = \frac{\sin\theta}{cos\theta}$, where $cos\theta \neq 0$	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370
3.8.B Describe key characteristics of the tangent function.	
3.8.B.1 Because the slope values of the terminal ray repeat every one-half revolution of the circle, the tangent function has a period of π.	Lesson 4.5, pp. 362-370
3.8.B.2 The tangent function demonstrates periodic asymptotic behavior at input values $\theta = \frac{\pi}{2} + k\pi,$ for integer values of k, because $cos\ =\ 0$ at those values.	Lesson 4.5, pp. 362-370
3.8.B.3 The tangent function increases and its graph changes from concave down to concave up between consecutive asymptotes.	Lesson 4.5, pp. 362-370
3.8.C Describe additive and multiplicative transformations involving the tangent function.	
3.8.C.1 The graph of the additive transformation $g\ (\theta)\ =\ tan\theta\ +\ d$ of the tangent function $f\ (\theta\ )\ =\ tan\theta$ is a vertical translation of the graph of f and the line containing its points of inflection by d units.	Lesson 4.5, pp. 362-370
3.8.C.2 The graph of the additive transformation $g(\theta) = tan(\theta + c)$ of the tangent function $f\ (\theta)\ =\ tan\theta$ is a horizontal translation, or phase shift, of the graph of f by $-c$ units.	Lesson 4.5, pp. 362-370
3.8.C.3 The graph of the multiplicative transformation $g\ (\theta)\ =\ atan\theta$ of the tangent function $f\ (\theta\ )\ =\ tan\theta$ is a vertical dilation of the graph of f by a factor of. If $a\ <\ 0$, the transformation involves a reflection over the x-axis.	Lesson 4.5, pp. 362-370

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3.8.C.4 The graph of the multiplicative transformation $g\ (\theta\ )\ =\ tan(b\ \theta)\ of$ the tangent function $f\ (\theta\ )\ =\ tan\theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\lvert\frac{1}{b}\rvert$. If $b\ <\ 0$, the transformation involves a reflection over the y-axis.	Lesson 4.5, pp. 362-370
3.8.C.5 The graph of $y = f(\theta) = atan(b(\theta + c)) + d$ is a vertical dilation of the graph of $y\ =\ tan\theta$ by a factor of $\lvert a\rvert$, has a period of $\lvert\frac{1}{b}\rvert\pi$ units, is a vertical shift of the line containing the points of inflection of the graph of $y\ =\ tan\theta$ by d units, and is a phase shift of $-c$ units.	Lesson 4.5, pp. 362-370
Topic 3.9 Inverse Trigonometric Functions	
3.9.A Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.	
3.9.A.1 For inverse trigonometric functions, the input and output values are switched from their corresponding trigonometric functions, so the output value of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function.	Lesson 4.7, pp. 378-386
3.9.A.2 The inverse trigonometric functions are called *arcsine*, *arccosine*, and *arctangent* (also represented as $sin^{-1}x$, $cos^{-1}x$, and $tan^{-1}x$). Because the corresponding trigonometric functions are periodic, they are only invertible if they have restricted domains.	Lesson 4.7, pp. 378-386
3.9.A.3 In order to define their respective inverse functions, the domain of the sine function is restricted to $[\ -\frac{\pi}{2},\frac{\pi}{2}\ ]$, the cosine function to $[0,\pi\ ]$, and the tangent function to $(-\frac{\pi}{2},\frac{\pi}{2})$	Lesson 4.7, pp. 378-386
Topic 3.10 Trigonometric Equations and Inequalities	
3.10.A Solve equations and inequalities involving trigonometric functions.	
3.10.A.1 Inverse trigonometric functions are useful in solving equations and inequalities involving trigonometric functions, but solutions may need to be modified due to domain restrictions.	Lesson 4.7, pp. 378-386; Lesson 5.1, pp. 395-402
3.10.A.2 Because trigonometric functions are periodic, there are often infinitely many solutions to trigonometric equations.	Lesson 4.7, pp. 378-386; Lesson 5.1, pp. 395-402
3.10.A.3 In trigonometric equations and inequalities arising from a contextual scenario, there is often a domain restriction that can be implied from the context, which limits the number of solutions.	Lesson 4.7, pp. 378-386; Lesson 5.1, pp. 395-402
Topic 3.11 The Secant, Cosecant, and Cotangent Functions	
3.11.A Identify key characteristics of functions that involve quotients of the sine and cosine functions.	
3.11.A.1 The secant function, $f(\ \theta)\ =\ sec\theta$, is the reciprocal of the cosine function, where $cos\theta \neq 0$.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370
3.11.A.2 The cosecant function, $f(\theta)\ =\ csc\theta,$ is the reciprocal of the sine function, where $sin\theta \neq 0$.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370
3.11.A.3 The graphs of the secant and cosecant functions have vertical asymptotes where cosine and sine are zero, respectively, and have a range of $(-\infty,-1] \cup [1,\infty)$.	Lesson 4.5, pp. 362-370

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3.11.A.4 The cotangent function, $f(\theta) = cot\theta$, is the reciprocal of the tangent function, where $tan\theta \neq 0$. Equivalently, $cot\theta = \frac{cos\theta}{sin\theta}$, where $sin\theta \neq 0$.	Lesson 4.2, pp. 327-335; Lesson 4.3, pp. 336-348; Lesson 4.5, pp. 362-370
3.11.A.5 The graph of the cotangent function has vertical asymptotes for domain values where $tan\theta = 0$ and is decreasing between consecutive asymptotes.	Lesson 4.5, pp. 362-370
Topic 3.12 Equivalent Representations of Trigonometric Functions	
3.12.A Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.	
3.12.A.1 The Pythagorean Theorem can be applied to right triangles with points on the unit circle at coordinates $(cos\theta, sin\theta)$, resulting in the Pythagorean identity: $sin^2\theta + cos^2\theta = 1$.	Lesson 5.1, pp. 395-402; Lesson 5.2, pp. 403-409
3.12.A.2 The Pythagorean identity can be algebraically manipulated into other forms involving trigonometric functions, such as $tan^2\theta = sec^2\theta - 1$, and can be used to establish other trigonometric relationships, such as $arcsinx = arccos(\sqrt{1 - x2})$, with appropriate domain restrictions.	Lesson 5.1, pp. 395-402; Lesson 5.4, pp. 418-429
3.12.B Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.	
3.12.B.1 The sum identity for sine is $sin(\alpha + \beta) = sin\alpha\, cos\beta + cos\alpha\, sin\beta$.	Lesson 5.3, pp. 411-417
3.12.B.2 The sum identity for cosine is $cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha\, sin\beta$.	Lesson 5.3, pp. 411-417
3.12.B.3 The sum identities for sine and cosine can also be used as difference and double-angle identities.	Lesson 5.3, pp. 411-417
3.12.B.4 Properties of trigonometric functions, known trigonometric identities, and other algebraic properties can be used to verify additional trigonometric identities.	Lesson 5.2, pp. 403-409; Lesson 5.3, pp. 411-417
3.12.C Solve equations using equivalent analytic representations of trigonometric functions.	
3.12.C.1 A specific equivalent form involving trigonometric expressions can make information more accessible.	Lesson 5.2, pp. 403-409; Lesson 5.3, pp. 411-417
3.12.C.2 Equivalent trigonometric forms may be useful in solving trigonometric equations and inequalities.	Lesson 5.1, pp. 395-402; Lesson 5.2, pp. 403-409; Lesson 5.3, pp. 411-417
Topic 3.13 Trigonometry and Polar Coordinates	
3.13.A Determine the location of a point in the plane using both rectangular and polar coordinates.	
3.13.A.1 The polar coordinate system is based on a grid of circles centered at the origin and on lines through the origin. Polar coordinates are defined as an ordered pair, (r, θ), such that $\lvert r \rvert$ represents the radius of the circle on which the point lies, and θ represents the measure of an angle in standard position whose terminal ray includes the point. In the polar coordinate system, the same point can be represented many ways.	Lesson 5.5, pp. 431-441
3.13.A.2 The coordinates of a point in the polar coordinate system, (r, θ), can be converted to coordinates in the rectangular coordinate system, (x, y), using $x = r\, cos\theta$ and $y = r\, sin\theta$.	Lesson 5.5, pp. 431-441
3.13.A.3 The coordinates of a point in the rectangular coordinate system, (x, y), can be converted to coordinates in the polar coordinate system, (r, θ), using $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan\left(\frac{y}{x}\right) \text{ for } > 0$ or $\theta = \arctan\left(\frac{y}{x}\right) + \pi \text{ for } x < 0$.	Lesson 5.5, pp. 431-441

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3.13.A.4 A complex number can be understood as a point in the complex plane and can be determined by its corresponding rectangular or polar coordinates. When the complex number has the rectangular coordinates (a, b), it can be expressed as $a + bi$. When the complex number has polar coordinates (r, θ), it can be expressed as $(r\, cos\theta\,) + i(r\, sin\theta)$	Lesson 2.4, pp. 190-200; Lesson 5.5, pp. 431-441
Topic 3.14 Polar Function Graphs	
3.14.A Construct graphs of polar functions.	
3.14.A.1 The graph of the function $r = f(\theta)$ in polar coordinates consists of input-output pairs of values where the input values are angle measures and the output values are radii.	Lesson 5.6, pp. 442-451
3.14.A.2 The domain of the polar function $r = f(\theta)$, given graphically, can be restricted to a desired portion of the function by selecting endpoints corresponding to the desired angle and radius.	Lesson 5.6, pp. 442-451
3.14.A.3 When graphing polar functions in the form of $r = f(\theta)$, changes in input values correspond to changes in angle measure from the positive x-axis, and changes in output values correspond to changes in distance from the origin.	Lesson 5.6, pp. 442-451
Topic 3.15 Rates of Change in Polar Functions	
3.15.A Describe characteristics of the graph of a polar function.	
3.15.A.1 If a polar function, $r = f(\theta)$, is positive and increasing or negative and decreasing, then the distance between $f(\theta)$ and the origin is increasing.	Lesson 5.6, pp. 442-451
3.15.A.2 If a polar function, $r = f(\theta)$, is positive and decreasing or negative and increasing, then the distance between $f(\theta)$ and the origin is decreasing.	Lesson 5.6, pp. 442-451
3.15.A.3 For a polar function, $r = f(\theta)$,, if the function changes from increasing to decreasing or decreasing to increasing on an interval, then the function has a relative extremum on the interval corresponding to a point relatively closest to or farthest from the origin.	Lesson 5.6, pp. 442-451
3.15.A.4 The average rate of change of *r* with respect to θ over an interval of θ is the ratio of the change in the radius values to the change in θ over an interval of θ. Graphically, the average rate of change indicates the rate at which the radius is changing per radian.	Lesson 5.6, pp. 442-451
3.15.A.5 The average rate of change of *r* with respect to θ over an interval of θ can be used to estimate values of the function within the interval.	Lesson 5.6, pp. 442-451
UNIT 4 Functions Involving Parameters, Vectors, and Matrices	
Topic 4.1 Parametric Functions	
4.1.A Construct a graph or table of values for a parametric function represented analytically.	
4.1.A.1 A parametric function in $\mathbb{R}^2$, the set of all ordered pairs of two real numbers, consists of a set of two parametric equations in which two dependent variables, x and y, are dependent on a single independent variable, t, called the *parameter*.	Lesson 1.5, pp. 105-113; Lesson 6.2, pp. 474-491

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4.1.A.2 Because variables *x* and *y* are dependent on the independent variable, *t* the coordinates (x_i, y_i) at time t_i can be written as functions of *t* and can be expressed as the single parametric function *f(t) = (x(t), y(t)),* where in this case *x* and *y* are names of two functions.	Lesson 1.5, pp. 105-113; Lesson 6.2, pp. 474-491
4.1.A.3 A numerical table of values can be generated for the parametric function *f(t) = (x(t), y(t))* by evaluating x_i and y_i at several values of t_i within the domain.	Lesson 6.2, pp. 474-491
4.1.A.4 A graph of a parametric function can be sketched by connecting several points from the numerical table of values in order of increasing value of *t*.	Lesson 6.2, pp. 474-491
4.1.A.5 The domain of the parametric function *f* is often restricted, which results in start and end points on the graph of *f*.	Lesson 6.2, pp. 474-491
Topic 4.2 Parametric Functions Modeling Planar Motion	
4.2.A Identify key characteristics of a parametric planar motion function that are related to position.	
4.2.A.1 A parametric function given by *f (t) = (x(t), y(t))* can be used to model particle motion in the plane. The graph of this function indicates the position of a particle at time *t*.	Lesson 6.2, pp. 474-491
4.2.A.2 The horizontal and vertical extrema of a particle's motion can be determined by identifying the maximum and minimum values of the functions *x(t)* and *y(t)*, respectively.	Lesson 6.2, pp. 474-491
4.2.A.3 The real zeros of the function *x(t)* correspond to *y*-intercepts, and the real zeros of *y(t)* correspond to x-intercepts.	Lesson 6.2, pp. 474-491
Topic 4.3 Parametric Functions and Rates of Change	
4.3.A Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.	
4.3.A.1 As the parameter increases, the direction of planar motion of a particle can be analyzed in terms of *x* and *y* independently. If *x(t)* is increasing or decreasing, the direction of motion is to the right or left, respectively. If *y(t)* is increasing or decreasing, the direction of motion is up or down, respectively.	Lesson 6.2, pp. 474-491
4.3.A.2 At any given point in the plane, the direction of planar motion may be different for different values of *t*.	Lesson 6.2, pp. 474-491
4.3.A.3 The same curve in the plane can be parametrized in different ways and can be traversed in different directions with different parametric functions.	Lesson 6.2, pp. 474-491; Lesson 6.3, pp. 492-502; Lesson 6.4, pp. 502-515
4.3.A.4 Over a given interval $[t_1, t_2]$ within the domain, the average rate of change can be computed for *x(t)* and *y(t)* independently. The ratio of the average rate of change of *y* to the average rate of change of *x* gives the slope of the graph between the points on the curve corresponding to t_1 and t_2, so long as the average rate of change of $x(t) \neq 0$.	Lesson 6.2, pp. 474-491
Topic 4.4 Parametrically Defined Circles and Lines	
4.4.A Express motion around a circle or along a line segment parametrically.	
4.4.A.1 A complete counterclockwise revolution around the unit circle that starts and ends at (*1, 0*) and is centered at the origin can be modeled by *(x(t), y(t)) = (cos t, sin t)* with domain $0 \leq t \leq 2\pi$.	Lesson 6.4, pp. 502-515

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4.4.A.2 Transformations of the parametric function $(x(t), y(t)) = (\cos t, \sin t)$ can model any circular path traversed in the plane.	Lesson 6.4, pp. 502-515
4.4.A.3 A linear path along the line segment from the point (x_1, y_1) to the point (x_2, y_2) can be parametrized many ways, including using an initial position $(x_1, y\ )$ and rates of change for x with respect to t and y with respect to t.	Lesson 6.2, pp. 474-491
Topic 4.5 Implicitly Defined Functions	
4.5.A Construct a graph of an equation involving two variables.	
4.5.A.1 An equation involving two variables can implicitly describe one or more functions.	Lesson 1.4, pp. 94-104; Lesson 6.2, pp. 474-491; Lesson 6.4, pp. 502-515
4.5.A.2 An equation involving two variables can be graphed by finding solutions to the equation.	Lesson 6.2, pp. 474-491
4.5.A.3 Solving for one of the variables in an equation involving two variables can define a function whose graph is part or all of the graph of the equation.	Lesson 6.2, pp. 474-491; Lesson 6.3, pp. 492-502; Lesson 6.4, pp. 502-515
4.5.B Determine how the two quantities related in an implicitly defined function vary together.	
4.5.B.1 For ordered pairs on the graph of an implicitly defined function that are close together, if the ratio of the change in the two variables is positive, then the two variables simultaneously increase or both decrease; conversely, if the ratio is negative, then as one variable increases, the other decreases.	Lesson 6.2, pp. 474-491
4.5.B.2 The rate of change of x with respect to y or of y with respect to x can be zero, indicating vertical or horizontal intervals, respectively.	Lesson 6.2, pp. 474-491
Topic 4.6 Conic Sections	
4.6.A Represent conic sections with horizontal or vertical symmetry analytically.	
4.6.A.1 A parabola with vertex (h, k) can, if $a \neq 0$, be represented analytically as $x - h = a(y - k)^2$ if it opens left or right, or as $y - k = a(x - h)^2$ if it opens up or down.	Lesson 6.3, pp. 492-502
4.6.A.2 An ellipse centered at (h, k) with horizontal radius a and vertical radius b can be represented analytically as $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. A circle is a special case of an ellipse where $a = b$.	Lesson 6.4, pp. 502-515
4.6.A.3 A hyperbola centered at (h, k) with vertical and horizonal lines of symmetry can be represented algebraically as $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ for a hyperbola opening left and right, or as $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$ for a hyperbola opening up and down. The asymptotes are $y - k = \pm\frac{b}{a}(x - h)$	Lesson 6.5, pp. 516-527
Topic 4.7 Parametrization of Implicitly Defined Functions	
4.7.A Represent a curve in the plane parametrically.	
4.7.A.1 A parametrization $(x(t), y(t))$ for an implicitly defined function will, when $x(t)$ and $y(t)$ are substituted for x and y, respectively, satisfy the corresponding equation for every value of t in the domain.	Lesson 6.2, pp. 474-491; Lesson 6.3, pp. 492-502; Lesson 6.4, pp. 502-515; Lesson 6.5, pp. 516-527

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4.7.A.2 If *f* is a function of *x*, then *y*= *f(x)* can be parametrized as *(x(t), y(t)) = (t, f (t))*. If *f* is invertible, its inverse can be parametrized as *(x(t), y(t)) = (f (t), t)* for an appropriate interval of *t*.	Lesson 1.5, pp. 105-113; Lesson 6.2, pp. 474-491
4.7.B Represent conic sections parametrically.	
4.7.B.1 A parabola can be parametrized in the same way that any equation that can be solved for *x* or *y* can be parametrized. Equations that can be solved for x can be parametrized as *(x(t), y(t)) = (f (t), t)* by solving for *x* and replacing *y* with *t*. Equations that can (be solved for *y* can be parametrized as *x(t), y(t)) = (t, f (t))* by solving for *y* and replacing *x* with *t*.	Lesson 6.2, pp. 474-491; Lesson 6.3, pp. 492-502
4.7.B.2 An ellipse can be parametrized using the trigonometric functions $x(t) = h + a\cos t$ and $y(t) = k + b\sin t$ for $0 \le t \le 2\pi$.	Lesson 6.4, pp. 502-515
4.7.B.3 A hyperbola can be parametrized using trigonometric functions. For a hyperbola that opens left and right, the functions are $x(t) = h + a\sec t$ and $y(t) = k + b\tan t$ for $0 \le t \le 2\pi$. For a hyperbola that opens up and down, the functions are $x(t) = h + a\tan t$ and $y(t) = k + b\sec t$ for $0 \le t \le 2\pi$.	Lesson 6.5, pp. 516-527
Topic 4.8 Vectors	
4.8.A Identify characteristics of a vector.	
4.8.A.1 A vector is a directed line segment. When a vector is placed in the plane, the point at the beginning of the line segment is called the tail, and the point at the end of the line segment is called the head. The length of the line segment is the magnitude of the vector..	Lesson 6.1, pp. 458-472
4.8.A.2 A vector $\underrightarrow{P_1P_2}$ with two components can be plotted in the *xy*-plane from $P_1 = (x_1, y_1)$ to P2 = (x2, y2). The vector is identified by *a* and *b*, where $a = x_2 - x_1$ and $b = y_2 - y_1$. The vector can be expressed as $\langle a, b\rangle$. A zero vector $\langle 0, 0\rangle$ is the trivial case when $P_1 = P_2$.	Lesson 6.1, pp. 458-472
4.8.A.3 The direction of the vector is parallel to the line segment from the origin to the point with coordinates *(a, b)*. The magnitude of the vector is the square root of the sum of the squares of the components.	Lesson 6.1, pp. 458-472
4.8.A.4 For a vector represented geometrically in the plane, the components of the vector can be found using trigonometry	Lesson 6.1, pp. 458-472
4.8.B Determine sums and products involving vectors.	
4.8.B.1 The multiplication of a constant and a vector results in a new vector whose components are found by multiplying the constant by each of the components of the original vector. The new vector is parallel to the original vector.	Lesson 6.1, pp. 458-472

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4.8.B.2 The sum of two vectors in $\mathbb{R}^2$ is a new vector whose components are found by adding the corresponding components of the original vectors. The new vector can be represented graphically as a vector whose tail corresponds to the tail of the first vector and whose head corresponds to the head of the second vector when the second vector's tail is located at the first vector's head.	Lesson 6.1, pp. 458-472
4.8.B.3 The dot product of two vectors is the sum of the products of their corresponding components. That is, $\langle a_1, b_1\rangle \langle a_2, b_2\rangle = a_1a_2 + b_1b_2$	Lesson 6.1, pp. 458-472
4.8.C Determine a unit vector for a given vector.	
4.8.C.1 A *unit* vector is a vector of magnitude 1. A unit vector in the same direction as a given nonzero vector can be found by scalar multiplying the vector by the reciprocal of its magnitude.	Lesson 6.1, pp. 458-472
4.8.C.2 The vector $\langle a, b\rangle$ can be expressed as $a\vec{i} + b\vec{j}$ in $\mathbb{R}^2$, where $\vec{i}$ and $\vec{j}$ are unit vectors in the x and y directions, respectively. That is, $\vec{i} = \langle 1, 0\rangle$ and $\vec{j} = \langle 0, 1\rangle$.	Lesson 6.1, pp. 458-472
4.8.D Determine angle measures between vectors and magnitudes of vectors involved in vector addition.	
4.8.D.1 The dot product is geometrically equivalent to the product of the magnitudes of the two vectors and the cosine of the angle between them. Therefore, if the dot product of two nonzero vectors is zero, then the vectors are perpendicular.	Lesson 6.1, pp. 458-472
4.8.D.2 The Law of Sines and Law of Cosines can be used to determine side lengths and angle measures of triangles formed by vector addition.	Lesson 5.4, pp. 418-429; Lesson 6.1, pp. 458-472
Topic 4.9 Vector-Valued Functions	
4.9.A Represent planar motion in terms of vector-valued functions.	
4.9.A.1 The position of a particle moving in a plane that is given by the parametric function $f(t) = (x(t), y(t))$ may be expressed as a vector-valued function, $p(t) = x(t)\underset{i}{\rightarrow} + y(t)\underset{j}{\rightarrow}$ or $p(t) = \langle x(t), y(t)\rangle$. The magnitude of the position vector at time *t* gives the distance of the particle from the origin.	Lesson 6.2, pp. 474-491
4.9.A.2 The vector-valued function $v(t) = \langle x(t), y(t)\rangle$ can be used to express the velocity of a particle moving in a plane at different times, *t*. At time *t*, the sign of *x(t)* indicates if the particle is moving right or left, and the sign of *y(t)* indicates if the particle is moving up or down. The magnitude of the velocity vector at time *t* gives the speed of the particle.	Lesson 6.2, pp. 474-491
Topic 4.10 Matrices	
4.10.A Determine the product of two matrices.	
4.10.A.1 An *n* × *m* matrix is an array consisting of *n* rows and *m* columns.	Lesson 7.1, pp. 534-545
4.10.A.2 Two matrices can be multiplied if the number of columns in the first matrix equals the number of rows in the second matrix. The product of the matrices is a new matrix in which the component in the *i*th row and *j*th column is the dot product of the *i*th row of the first matrix and the *j*th column of the second matrix.	Lesson 7.1, pp. 534-545

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Topic 4.11 The Inverse and Determinant of a Matrix	
4.11.A Determine the inverse of a 2 × 2 matrix.	
4.11.A.1 The identity matrix, *I*, is a square matrix consisting of 1s on the diagonal from the top left to bottom right and 0s everywhere else.	Lesson 7.1, pp. 534-545
4.11.A.2 Multiplying a square matrix by its corresponding identity matrix results in the original square matrix.	Lesson 7.1, pp. 534-545
4.11.A.3 The product of a square matrix and its inverse, when it exists, is the identity matrix of the same size.	Lesson 7.1, pp. 534-545
4.11.A.4 The inverse of a 2 × 2 matrix, when it exists, can be calculated with or without technology.	Lesson 7.1, pp. 534-545
4.11.B Apply the value of the determinant to invertibility and vectors.	
4.11.B.1 The determinant of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$. The determinant can be calculated with or without technology and is denoted det(A).	Lesson 7.1, pp. 534-545
4.11.B.2 If a 2×2 matrix consists of two column or row vectors from $\mathbb{R}^2$, then the nonzero absolute value of the determinant of the matrix is the area of the parallelogram spanned by the vectors represented in the columns or rows of the matrix. If the determinant equals 0, then the vectors are parallel.	Lesson 7.1, pp. 534-545
4.11.B.3 The square matrix A has an inverse if and only if det$(A) \neq 0$.	Lesson 7.1, pp. 534-545
Topic 4.12 Linear Transformations and Matrices	
4.12.A Determine the output vectors of a linear transformation using a 2 × 2 matrix.	
4.12.A.1 A linear transformation is a function that maps an input vector to an output vector such that each component of the output vector is the sum of constant multiples of the input vector components.	Lesson 7.2, pp. 546-555
4.12.A.2 A linear transformation will map the zero vector to the zero vector.	Lesson 7.2, pp. 546-555
4.12.A.3 A single vector in $\mathbb{R}^2$ can be expressed as a 2×1 matrix. A set of n vectors in $\mathbb{R}^2$ can be expressed as a $2 \times n$ matrix.	Lesson 7.2, pp. 546-555; Lesson 7.3, pp. 556-564
4.12.A.4 For a linear transformation, L, from $\mathbb{R}^2$ to $\mathbb{R}^2$, there is a unique 2×2 matrix, A, such that $L(\vec{v}) = A\vec{v}$ for vectors in $\mathbb{R}^2$. Conversely, for a given 2×2 matrix, A, the function $L(\vec{v}) = A\vec{v}$ is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$.	Lesson 7.2, pp. 546-555; Lesson 7.3, pp. 556-564
4.12.A.5 Multiplication of a 2×2 transformation matrix, A, and a $2 \times n$ matrix of n input vectors gives a $2 \times n$ matrix of the n output vectors for the linear transformation $L(\vec{v}) = A\vec{v}$.	Lesson 7.2, pp. 546-555; Lesson 7.3, pp. 556-564
Topic 4.13 Matrices as Functions	
4.13.A Determine the association between a linear transformation and a matrix.	
4.13.A.1 The linear transformation mapping $\langle x, y \rangle$+ to $\langle a_{11}x + a_{12}y, a_{21}x + a_{22}y \rangle$ is associated with the matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.	Lesson 7.3, pp. 556-564

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4.13.A.2 The mapping of the unit vectors in a linear transformation provides valuable information for determining the associated matrix.	Lesson 7.3, pp. 556-564
4.13.A.3 The matrix $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is associated with a linear transformation of vectors that rotates every vector an angle θ counterclockwise about the origin.	Lesson 7.3, pp. 556-564
4.13.A.4 The absolute value of the determinant of a 2×2 transformation matrix gives the magnitude of the dilation of regions in $\mathbb{R}^2$ under the transformation.	Lesson 7.3, pp. 556-564
4.13.B Determine the composition of two linear transformations.	
4.13.B.1 The composition of two linear transformations is a linear transformation.	Lesson 7.3, pp. 556-564
4.13.B.2 The matrix associated with the composition of two linear transformations is the product of the matrices associated with each linear transformation.	Lesson 7.3, pp. 556-564
4.13.C Determine the inverse of a linear transformation.	
4.13.C.1 Two linear transformations are inverses if their composition maps any vector to itself.	Lesson 7.3, pp. 556-564
4.13.C.2 If linear transformation, L, is given by $L(\vec{v}) = A\vec{v}$, then its inverse transformation is given by $L^{-1}(\vec{v}) = A^{-1}\vec{v}$, where is the inverse of the matrix A.	Lesson 7.3, pp. 556-564
Topic 4.14 Matrices Modeling Contexts	
4.14.A Construct a model of a scenario involving transitions between two states using matrices.	
4.14.A.1 A contextual scenario can indicate the rate of transitions between states as percent changes. A matrix can be constructed based on these rates to model how states change over discrete intervals.	Lesson 7.4, pp. 565-573
4.14.B Apply matrix models to predict future and past states for *n* transition steps.	
4.14.B.1 The product of a matrix that models transitions between states and a corresponding state vector can predict future states.	Lesson 7.4, pp. 565-573
4.14.B.2 Repeated multiplication of a matrix that models the transitions between states and corresponding resultant state vectors can predict the steady state, a distribution between states that does not change from one step to the next.	Lesson 7.4, pp. 565-573
4.14.B.3 The product of the inverse of a matrix that models transitions between states and a corresponding state vector can predict past states.	Lesson 7.4, pp. 565-573